Pythagorean Theorem Proofs and Presentations:
Refer to Website for Diagrams and text as needed
25 August 2008

(please excuse any mistyped names)

First we discussed Mary Katherine's proof (3 others used the same proof), which is actually Bhaskara's proof and dates to the second century AD. The proof was based on tessellating/tiling a right triangle or transforming it in such a way that four such triangles form a large square with a smaller square in the middle. We find the area of the large square by adding the area of the small square and the 4 triangles (see the proofs on the site). By algebraic manipulation on the components of the area formulas we show the Pythagorean theorem to be valid.

Questions arose, with a couple of reminders from Dr. McCrory:
First, when pulling sources from outside, such as the internet, be sure to cite them.

Second, do people understand the proof and how it works?

Dr. McCrory suggested care in labeling diagrams. In particular, when a theorem generally uses specific variables, as is the case for a, b and c in Pythagoras, it is best to use the convention in proofs and work.

Cut the knot dot com was mentioned as a source for the information, and Dr. McCrory suggested this as a good source.

Finally, we noted that Mary Katherine judiciously used labels, which helped keep the construction free of clutter.

Finally, we had some discussion of how to create 4 congruent triangles by rotation in GSP. We discussed using the rotations tool from the construct menu as an alternative, albeit a less rigorous one than either translating by vector (select a point on the figure you want to move, map it to a point where you want it to end up, mark vector in the transform menu then highlight what you want to move and translate by vector –this is more accurate and more of a construction technique than manipulating with the rotator tool.

Ginny chose to use a similar proof that has been attributed to Pythagoras.
She spoke about using rid paper to guide her construction. This allowed her to be able to plot the points in her diagram then connect them by segments. (We discussed that this was not technically a construction.)

In understanding the construction we discussed that in this case the area of the large square, as before, was the area of the small square plus the areas of the 4 triangles contained in the large square.

In discussing the work, we found the use of labels, color, and larger text to be particularly helpful.

Questions: Can you fill in the shapes? How. GSP directions: Select the vertices of a polygon, CONSTRUCT polygon interior-GSP will fill it in. DISPLAY color to change the appearance of the shaded area.

Adam presented the Garfield Proof (2 others did this proof). The proof has been attributed to President Garfield, who worked on this before his term as President. This proof made use of a right trapezoid divided into 3 right triangles, 2 of which are congruent, instead of a square. It turns out that this proof is related to Ginny's, only using _ of the figure that Ginny used.

Of particular note in Adam's document was the fact that he used multiple pages instead of putting several drawings on one page. To do this, FILE-Document Options, then add, remove, rearrange, and label pages as desired—tabs will show at the bottom edge of the document.

There was a GSP Question about labeling lines. You can do this in essentially the same manner as labeling points.

Final Proof: Similar triangles (I didn't get the presenter's name). Approximately _ the class used this proof. It was suggested that this proof was used by Legendre. It is different from the other proofs in that it is strictly algebraic and doesn't take advantage of area concepts. The Ancient Greeks could not have used this proof as the algebra was not developed in their time.

There were almost immediate questions about this proof, including: how do you know which triangles are similar? Which triangles to compare? Where the triangles come from and how you can use them?

In this case we began with a right triangle, and by dropping an altitude from the vertex of the right angle, we created 2 triangles similar to the first. By analyzing the triangle and seeing some angle relationships, it became apparent pretty quickly which triangles to use.

Dr. McCrory pointed out the importance of similarity in Geometry and that there is a need to start understanding it immediately.

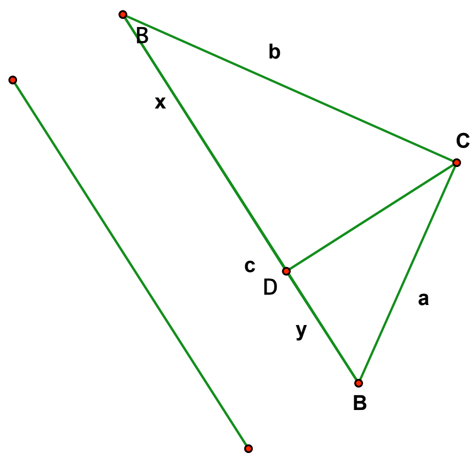
We discussed hiding items in sketchpad and gave some directions. First, you could highlight what you want to hide, then DISPLAY-Hide whatever.

Somewhere we started to talk about readability of constructions and considerations of labels, colors, text, etc. We also briefly mentioned including descriptive text on constructions in text boxes.

## Introduction:
## Similar Triangles

Question: What does it mean for triangles to be similar?
And a reminder from Dr. McCrory to be precise in descriptions and definitions…it is more than just congruent angles and proportional sides.
PRECISELY, triangles are similar when corresponding angles are equal (congruent) and the ratios of corresponding sides are proportional.

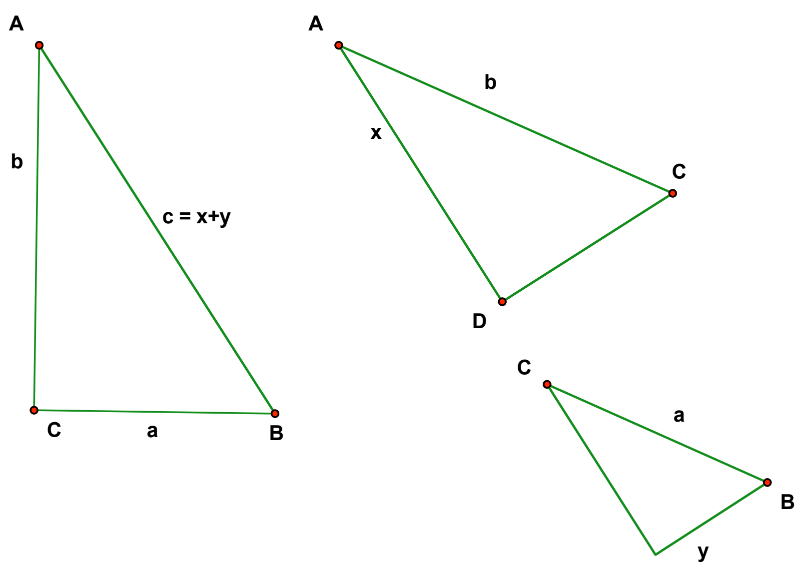
To prove triangles similar, we will only need to know that 2 pairs of angles have equal measure since the angles sum to 180 in all triangles.
There are 6 possible conditions for similarity between 2 triangles. If triangles ABC and A'B'C' are similar, then:
The following pairs of angles must be equal:  A and A'; B and B'; and C and C'. Additionally, the sides must be in a constant ratio, *i.e.*,  AB:A'B' = AC:A'C' = BC:B'C'.




Construct right triangle ACB, wth right angle ACB.  Construct an altitude from C to AB at D.  I have attempted to rotate the triangles into similar orientation with the right angle in the bottom left corner of the figures.

For triangles ACB and ADC, note that angles C and D are both right, thus congruent. Angle A is congruent to itself. Angles A and B must be congruent since 2 angles of 2 triangles are congruent and all 3 sum to 180, the 3rd angle pair must also be congruent. Thus by Angle Angle Angle, the triangles are congruent.

For Triangles ADC and CDB, we know that CD is the altitude of triangle ACB thus both angles at D are right angles and angle ADC is congruent to angle CDB. Angle ACB is right, thus angles BCD and ACD are complementary. Also, anges A and B are complementary. However, angles A and ACD are also complementary, since they are the non-right angles of a right triangle. Thus, angles A and A and BCD are congruent since they are both complementary to the same angle. By similar argrument, angles B and ACD are comlpementary, and Trianges ADC and CDB are similar by the Angle Angle Angle theorem as well.



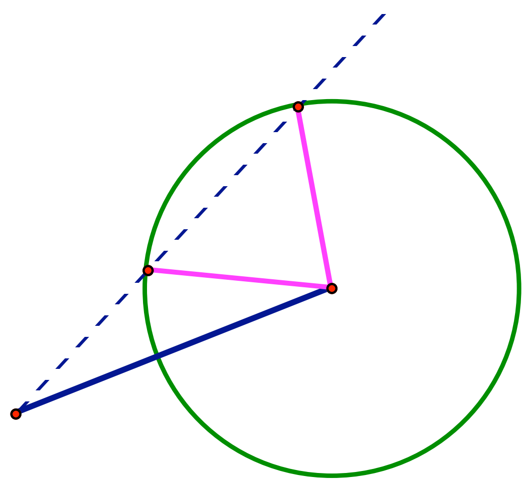

For triangles to be similar, all equations must work. However, there are a variety of ways in which  3 conditions will imply all six, and we will discuss them in detail over the next several classes: specifically, knowing

1)  ANGLE ANGLE ANGLE (which is the same as ANGLE ANGLE since the 3$^{rd}$ angle must be supplementary to the other two combined); ANGLE SIDE ANGLE and ANGLE ANGLE SIDE (as long as you have 2 angles you have everything else.  If you put in a side, it serves as a reminder of the similarity ratio)

2)  SIDE ANGLE SIDE: knowing 2 sides and an included angle gives us sides of a specific length that form the angle. There will be only 1 way to connect the non-included vertices of the two segments.

3)  SIDE SIDE SIDE: if the side ratios are the same, then the angle ratious must be as well.

NOTE: ANGLE SIDE SIDE does not work—it could lead to 2 different triangles.



Again, a reminder from Dr. McCrory that similarity is the most useful tool in plane geometry.

Construction Homework:
Doing constructions with GSP
27 August 2008

Questions about the homework:
Summarizing, for now…
Constructions of various shapes can be built on each other or done separately

Whether or not to hide construction lines is a personal decision. The form of the work isn't as important as the mathematics. However, GSP files must always be turned in.

Constructions are just that…working from a grid is not strictly part of a construction. The original input must be in the form of a line segment (for this assignment)

The dilation and rotation tools are not part of true constructions.

ALL POLYGON CONSTRUCTIONS FOR THE ASSIGNMENT BEGIN WITH AN EDGE OF THE POLYGON TO BE CONSTRUCTED

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Doing constructions with GSP
MUST USE THE CONSTRUCT MENU

For now we will be using the construct menu minus "locus". A warning to be careful with "point on object" as it constructs a random point, and as a rule we will be working with endpoints, intersections, and vertices.

Dr. McCrory discussed the historical background of constructions, beginning with a problem posed by the Greeks before Euclid: What can you draw if you only have an arbitrarily long ruler (lines) and compass (circles)? Note: in the case of constructions, a ruler refers to a straightedge—there is no measurement allowed in constructions. In this course, however, measurement is going to be important for educational purposes as well as being a connection to science.

Straightedge construction: lines. Rays and segments are variations of lines.

Compass constructions: Circle by center and point, which is particularly useful for constructing intersections.

All other menu items are derived from the above.

We can always construct a line through any 2 points. Similarly, we can construct intersections where 2 lines meet in a point.

Measurement is not part of construction, but it can be used as a way to verify the work we do. This is particularly true due to the accuracy of GSP as a measuring tool.
Remember…measurement is not part of a construction and a construction is not a proof, nor

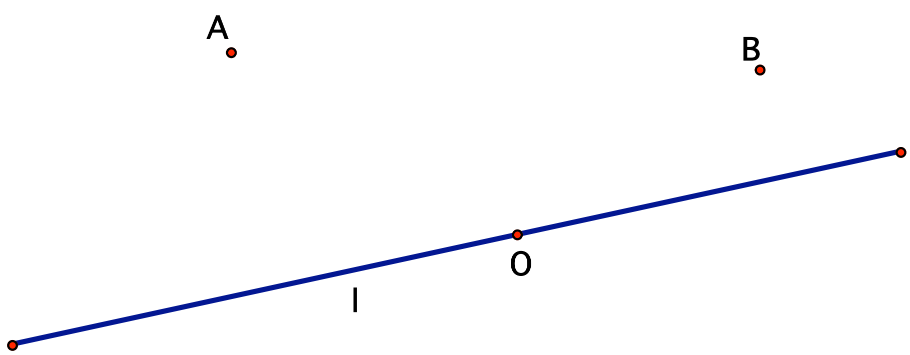
do you need to prove that constructions work. For more specifics on constructions, refer to the course website and Dr. McCrory's notes.

From the homework:
#1. Equal refers to having the same measure, i.e. numerical value, as opposed to congruent. There is a technical difference here, however, Dr. McCrory may use the terms interchangeably since it is more understandable.

Also, note the definition of regular in considering constructions.

#2. Question to consider: What do you know about the proportions between s and s'? How must the sides compare in order for the new square to have area double that of the original? This problem is equivalent to: how do you construct a segment of the necessary length to be able to construct the square or appropriate area.

Another problem to think about that is a more advanced construction problem (extra credit): Given a line L and 2 points A and B not on the line, how do you construct a point O on line L s.t. angle AOB measures 45 degrees? (Hint: use the inscribed angle theorem—this goes back to the relationship between and angle inscribed in the circle and the angle of the subtended arc.).



# Review of Constructions
# And
# Information on Polygon Constructions and
# Construction of Geometric Means
29 August 2008

Unlike many days, there was no clear statement today about a topic of the day. However, after opening the floor for constructions, Dr. McCrory took a significant chunk of the class to discuss the last assignment and what he was looking for in the constructions. We also discussed how to check constructions to see how they were done without looking at the script from creating a tool (which we will discuss next week).

We began class discussing the homework due Friday morning. There was a question about geometric means (which we'll get to). First, consider the Polygon constructions and be aware of the various constructions we did NOT do in this batch, essentially those with an odd number of sides—pentagon, heptagon, and nonagon.

We first looked at Julia's Octagon construction. There was a problem in that she didn't begin with the starting point that was given—a side of the octagon. In fact, she did not remember how she did the construction other than by trial and error, so Dr. McCrory began the process of deconstructing and figuring out how the construction was performed without looking at the script for the construction. This was particularly useful as it let us see first of all the interactions between constructed points and objects with the final construction—there were clear differences between say input points and objects dependent on those points.

After considerable investigation with the construction, it became clear that one of the keys to construction is knowing something about the figure to be constructed AND the need to follow directions.

Further example of this…consider the construction of a segment whose length is the geometric mean of the segments. Go back and reread the question and note that between the hint and the original problem, we're told we have: two segments, one of length x, the other length y, and the hint suggesting using the diameter of a circle. There are 2 more or less correct ways to approach the problem. The first, and more correct, is to start with 2 independent segments x and y, then to put one of them on a line and construct the other segment on the same line with both segments sharing an endpoint (say AB is length x and BC is length y). From there, proceed with the construction by finding the midpoint, using that and either A or C to construct a circle by center and point on circle. Construct a perpendicular through B and find the point D where that intersects the circle. The intersection becomes the vertex of a right triangle, and BD is an altitude from to the hypotenuse. Alternatively, you could start with a diameter and construct the point D on the diameter through which the altitude runs, then continue as above. The two segments will not be as independent as in the first version of the construction, but neither of the segments will have fixed length on the diameter.

This led to a long discussion of "correct input" with the reinforced reminder that "given" means that's what you have to start with, nothing more or less except for construction tools.

Remember:  GSP records everything you do, and this can be sued to check your work. You can see what the script was by creating a told and showing script view—everything about the line in script view is what GSP considers as given.

Tools, as we learn to use them, allows up to save constructions that we may want to use later to help in a bigger construction.

There will be more constructions for Wednesday. Remember to give explanations about why the proof works. (editorial comment: I learned a couple of years ago to keep a text box open while doing constructions and write what I'm doing as I go. This way I don't need to try and reconstruct my work later as I have it recorded as I work. If I explain what I'm doing as I go, it gets much easier to both remember what I did and see how the work might be useful at a later date. erg)

A reminder from Dr. McCrory that we will eventually be shifting to proof, and that we should remember to be aware that proofs and constructions are somewhat analogous. We were reminded that in the "Elements", Euclid reminded us that construction and proof go hand in hand.

Note from Dr. McCrory: a good graduate student problem/project would be to discuss the problem of constructing regular polygons, which are related to abstract in some Gaussian breakthrough.

As far as the new homework is concerned:
Trisecting a segment hint: use similar triangles.
A reminder that trisecting a segment is a special case of b—dividing a segment into n equivalent segments.

2—keep track of ratios and what length ratio you need.