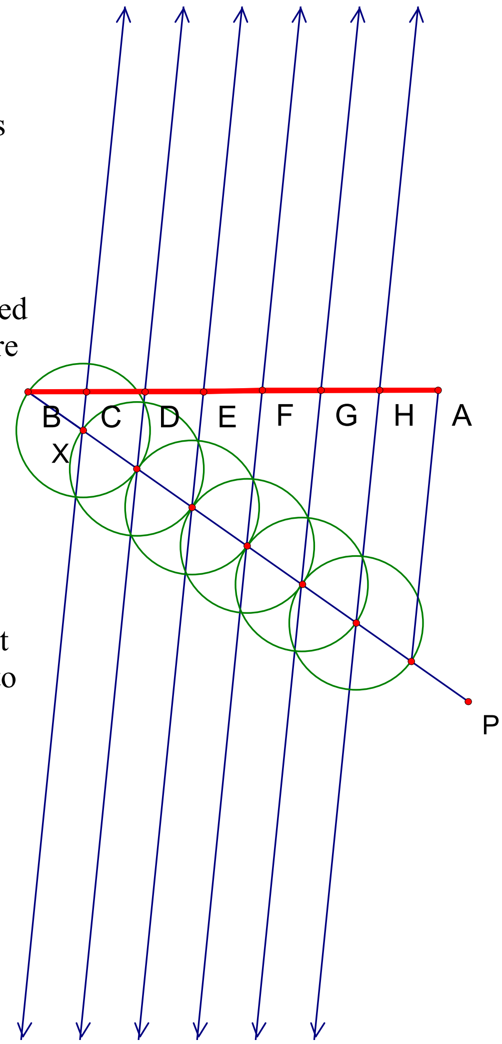
Math 5200/7200 Geometry
Class Notes

Note to class: // is my symbol for parallel, ~ for similar

<u>Wednesday, September 3, 2008</u>

Laura Hill explains homework question 1

McC: Shows hidden, she starts with segment AB, She did it right since as points A and B move the Segment AB is still divided into 7 equal parts. You see she did it right since points move and stay equal



Laura: Use properties of // lines
Made segment AB, put point P not on AB, constructed segment BP, plotted point X on BP, constructed circle centered at X, thru point B, made 5 more congruent circles along segment BP. The last point on segment BP was connected to A. Then // lines were constructed thru all the points on BP that are // to the line thru point A.

Reminder: To make tool, select your diagram, go to the Tool menu the double arrows at the bottom of the tool bar, create new.

McC's suggestion: Instead of making segment BP, make a ray. If you put point to close to point P on segment BP you may not have enough space to mark off the 7 congruent segments. Also to get the first point on the ray, instead of just putting the point there make a circle centered at B and construct the intersection of that circle and the ray.
Do you understand? Why does it work?
What properties of // lines did you use?

Laura: n is 7 so you make n – 1 circles, connect last point and make the // lines. Similar angles…Michelle adds corresponding angles

McC: All segments you made are congruent, if all here congruent how are they congruent on other side?
Laura shows examples of ~ triangles. (there are 7—do you see them all?)

Here's an important theorem: If parallel lines are cut by transversals, those transversals divide the lines proportionally.

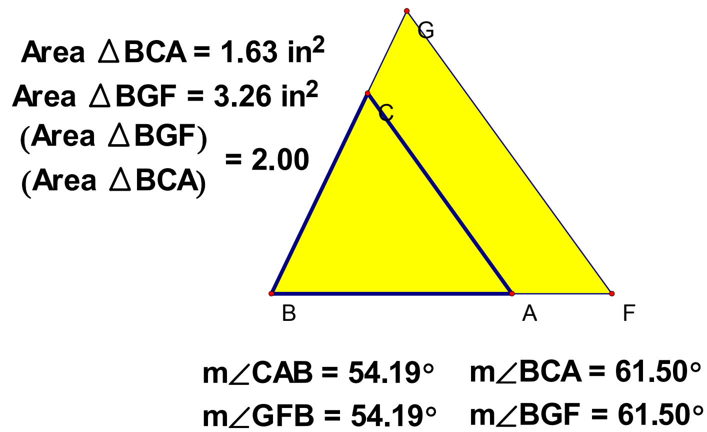
Adam: I didn't use // lines at all….

McC: Yes there's another way:
We look at Adam's picture which I don't have access to, so any attempt of mine to explain it will be lost. (sorry I can't do it justice Adam, but it was very cool.) Here's what I caught in class: Midpoint of the given segment, made squares the size of half the given segment, that gives a 1 by 2 rectangle, then things with diagonals (this is where it was hard to follow)—two diagonals with rectangles, diagonals of squares, their intersection gives third and a perpendicular dropped from that point to the original segment marks off a third of the segment.

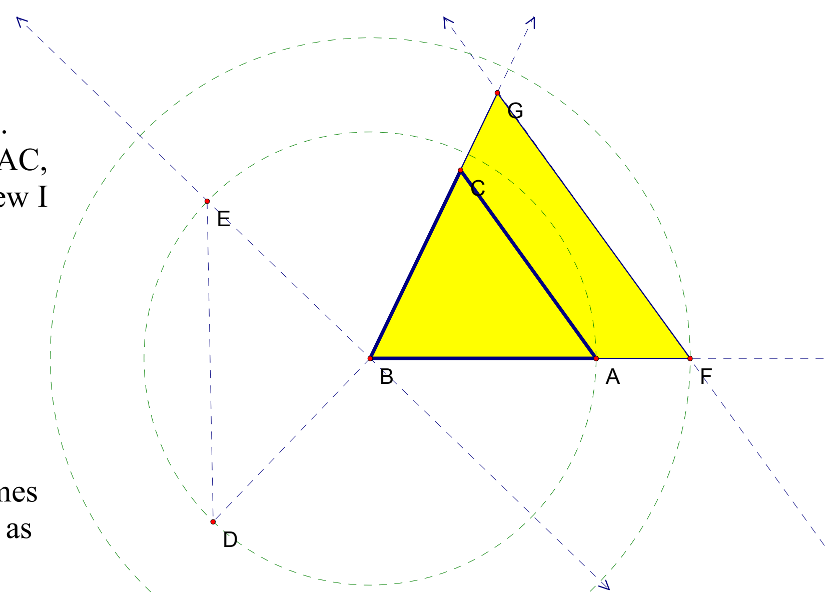
McC: In doing 1/7 constructs harmonic series (something we should remember from 3200)Not as easy to see why this works. This is too interesting, so it will take a lot of class time (more than we have) to explain. The idea is that it's an inductive procedure, third, to fourth, to fifth, to sixth, to seventh.
However, if you play with GSP enough, you can stumble upon this.

Russ presents homework question 7

Given ΔABC construct a second triangle so that the ratio of areas is 2, move the points and the ratio stays the same and the angles stay the same,
similarity ratio is 1.41



Show all hidden
Russ: Similarity ratio is 1.41 found that playing which gsp, so area ratio would be 2.
That 1.41 is root 2 –I know from seeing it all the time.
Made 1st triangle, then constructed circle with radius AC, then right triangle, ΔEBD to get hypotenuse ED—knew I needed hypotenuse not sure why ---



Questions from class:
How did you get the larger circle?
Using ED as radius and B as center.

What is special about ED? About that right triangle?
It's isosceles…*So what?*, The hypotenuse is root 2 times leg, then larger circle is center a with that hypotenuse as radius

How did you find point F?
Ray from B to A where ray intersects bigger triangle other leg of triangle must be // to third leg of original.

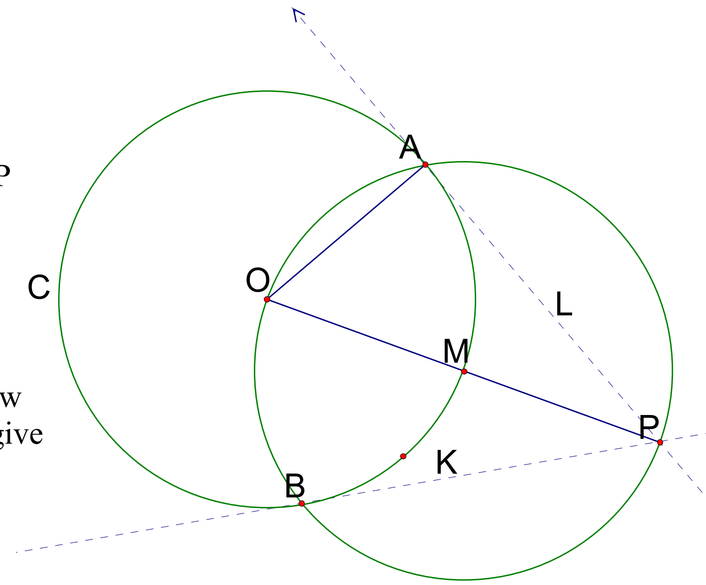
How did you have point b? How did you know where BC is?
That's given
Ray AC, where larger circle interested triangle (*I got this statement form class, but looking at the picture it doesn't make sense.)*

McC: Technically speaking point on object isn't a construction available to us, but I'mnot making a big deal
This is especially nice drawing because, but this way the auxiliary triangle isn't overlapping the original. That similar triangle can be anywhere and this is the most convenient way to do it, constructing new on top of old.
This is very important b/c emphasizes the importance of area of similarity.

Another big theorem: If length ratio is r, then area ratio is r^2, so if area ratio is s, then length ratio is root $\sqrt{s}$

McC: This is key example in course, area is key and sadly neglected in school mathematics, volume also.

Charnelle explains 3



Start with circle, point P
Segment OP
Then midpoint of OP, call it M, circle with M as center thru P and O.
Select intersections of two circles. Draw perpendicular to those points to get tangent line,
Yes, perpendicular went thru point p.

McC: What's snother way to connect point to P and you know its perp. Why did this work? How did you know this would give you tangent?

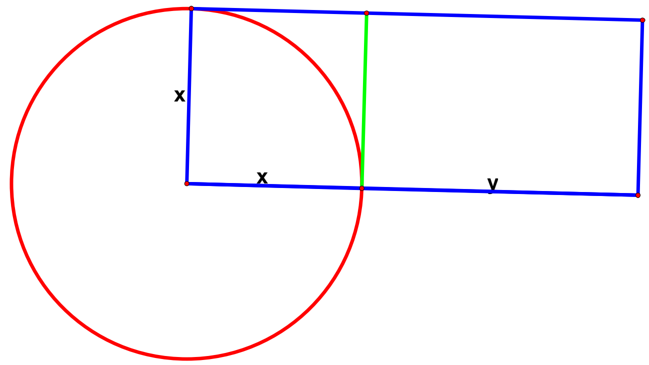
Charnelle: Any line perpendicular to radius is tangent to the circle.
McC: How do you know the line is perpendicular? When you constructed perpendicular, how did you know it would go thru point H?
Charnelle: These segments are equidistant to circle.
McC: How did you know it was right triangle? How is hypotenuse related to the circle?
Charnelle: Its diameter.
McC: We have theorem: If a triangle inscribe in a circle has one side as the diameter, then the angle opposite it is right angle—not the last time you'll see this- ***<u>one of his favorite theorems!!!</u>***
This is good way to make right tri given hypotenuse.
Central angle is twice the inscribed angle—so we know this theorem as special case
So if writing proof you can use the original theorem or this corollary.

McC: Weak point is that many people didn't explain why their construction worked.
There's not any big difference, only a difference of degree in explaining why a construction works and proving, won't make a big distinction between these two at this point in the class.

Explain why works based on other stuff you already know—problems are constructed so that you can use what we've already done in this course.
Proofs not in specific form, just very clear explanation using facts we already discussed or you already know.

McC: Golden rectangle—who knows what it is
Charnelle: Diagonal of rectangle to side is 1 to 5/3 it
involves golden ratio and diagonal
McC: Not diagonal, sides. Base to height is golden ratio, but what is golden ratio?
Ancient Greeks had properties of this rectangle involving proportions.
Cut a square off what's left is another golden rectangle, has same proportions as what you started with,

In algebra, which the Greeks didn't have, but we do (lucky us ☺)
Height is x, length is $x + y$. How can we say large and small have same ratio as long to little side?



$\frac{x}{y} = \frac{x+y}{x}$, pretty to do construction repeatedly—way to construct logarithmic spiral

Some algebra: phi φ (say fee) is golden ratio symbol

$$\varphi = \frac{x}{x} + \frac{y}{x}$$

$$\varphi = 1 + \frac{y}{x}$$

$$and \quad \frac{y}{x} = \left(\frac{x}{y}\right)^{-1} \quad so \quad \frac{y}{x} = \varphi^{-1} = \frac{1}{\varphi}$$

$$\varphi = 1 + \frac{1}{\varphi}$$

$$\varphi^2 = \varphi + 1$$

$$\varphi^2 - \varphi - 1 = 0$$

$$\varphi = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(-1)}}{2(1)}$$

$$\varphi = \frac{1 \pm \sqrt{1+4}}{2} = \frac{1 \pm \sqrt{5}}{2}$$

$$\varphi = \frac{1 + \sqrt{5}}{2}$$

Since we're dealing with distance, we only need positive.

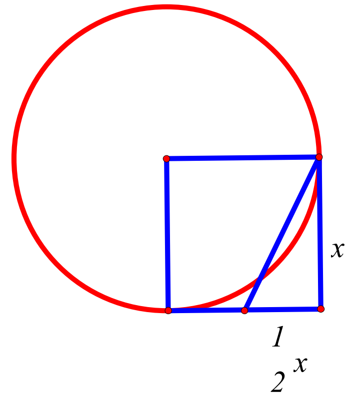
McC: way we construct uses diagonal of something so maybe this is what Charnelle remembers.

notice interesting $\varphi = 1 + \frac{1}{\varphi}$, $\frac{1}{\varphi}$ is 0.618

Greeks used this ratio in their architecture **popular term project** (lots of examples of the golden ratio in nature too)

Wikipedia good resourceone guy who updates math Wikipedia—that's why they are good

Greeks could construct this golden ratio— but we'll draw since time limited,…
Start with short side, and create long side. Construct 1 + root 5 over two using Pythagorean theorem.
Start with short side, construct square,
construct midpoint of one side,
connect midpoint to vertex square to create right triangle.



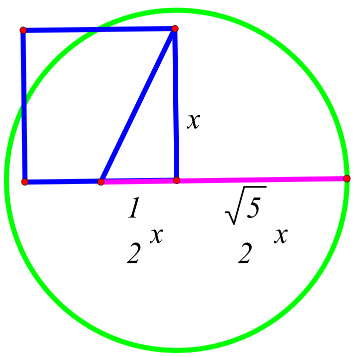
$$\left(\frac{1}{2}x\right)^2 + x^2 = h^2$$

$$\frac{1}{4}x^2 + x^2 = h^2$$

$$\frac{5}{4}x^2 = h^2$$

$$\frac{x\sqrt{5}}{2} = h$$

The hypotenuse is the length we need. Construct a circle that hypotenuse as radius and center as vertex of right angle of right triangle. Then our pink segment is the golden ratio.



<u>Notes on Friday, September 5, 2008</u>

Remember to check the website for notes and such,
Let's discuss next week: Here are graduate students, pick groups, 20 minute presentation
We are making the transition from constructions to proof.
How you present is your decision- better if not grad students, not everyone has to present, need a write up for this, Email files by class time!

General discussion of all projects
<u>1st project</u>—show how to do all the other constructions (midpoint, parallel line, perpendicular line, angle bisector, circle by center and radius) on the construct menu—using only what the Greeks had a available intersection, line/ray/segment, circle by center and point

<u>2nd project</u> —Construct a regular pentagon, use fact that the ratio of diagonal to an edge is the golden ratio to do this construction, extra credit is to figure how why diagonal to edge is the golden ratio—you won't have time to present both of these.

<u>3rd and 4th projects</u> —Properties of Quadrilaterals, what are the definitions—use only most basic properties form which other properties can be derived. Definition should be reflected in the word used to name the object, parallelogram quadrilateral with opposite sides parallel, theses two groups are connected so they will need to communicate, **Rectangle** is quadrilateral where adjacent sides are perpendicular, **Rhombus** is quadrilateral with all four sides equal, **Square** is a regular quadrilateral, **Kite** has 2 pairs of adjacent congruent sides---Different

Geometry books may do these definitions differently—AS A TEACHER YOU NEED TO BE AWARE OF THIS!!!!!!!!!!!!!

Some inclusions are automatic, others will have to be proven. i.e. Show a rectangle is a parallelogram.

Prove relation between the sides of the parallelogram.
– Opposite sides have same length, converse is true also.
Quadrilateral is parallelogram if and only if opposite sides are equal.
Then analyze properties of diagonals—(lines that connect opposite vertices)
Quadrilateral is parallelogram if and only if diagonals bisect each other.
Quadrilateral is rectangle if and only if diagonals are congruent.
Proofs can be informal, but need to use diagrams. May not be able to do all the proofs in class so you will have to choose.

Focus on diagonals for rhombus, square, and kite:
Diagonals of rhombus bisect the angles, and are perpendicular.
Diagonals of a kite are perpendicular, and only one is bisected
(not the same size unless a kite is a rhombus)

5^th^ and 6^th^: Concurrence theorems for Triangles, Concurrent lines are lines that go through the same point.

In general you should ask for the proofs, "what can you use"?
Basic facts, list separately what facts you have used.
State the theorem used, so we will accumulate a list of basic theorems used.
This sounds like it would be a good study tool for he first test.