

## MATH 5210/7210 Exam 2

Take-home exam, due Tuesday, April 15, at 11:00 am

You may turn in a hard copy or an electronic copy of your exam. You may also turn in models.

*Do not consult with anyone other than Dr. McCrory.* You may use your course notes or the course web site. Do not look up answers in books or on the internet.

1. Describe how to fill space with regular tetrahedra and regular octahedra (a 3-dimensional tessellation). As part of your description, answer the following questions:

(a) Can two tetrahedra share a face? Can two octahedra share a face?

(b) How many octahedra and how many tetrahedra share each edge? Describe the arrangement of tetrahedra and octahedra around each edge. Is there more than one type of edge?

(c) How many octahedra and how many tetrahedra share each vertex? Describe the arrangement of tetrahedra and octahedra around a vertex. Is there more than one type of vertex?

2. (a) Describe the symmetry group of a rectangle. Assume the rectangle is not a square. In other words, the rectangle has base  $b$  and height  $h$  with  $b \neq h$ . List all the symmetries of the rectangle, and describe each symmetry geometrically. Give the multiplication table of the symmetries of the rectangle.

(b) Describe the symmetry group of a box. (A box is a polyhedron with six rectangular faces and all dihedral angles right angles.) Assume the box has no square sides. In other words, the box has length  $l$ , width  $w$ , and height  $h$ , with  $l \neq w$ ,  $l \neq h$ , and  $w \neq h$ . List all the symmetries of the box, and describe each symmetry geometrically. Give a complete geometric description of the multiplication of the symmetries of the box. (You do not have to write out the whole table.)