MATH 4010/6010 Final Exam Solution Guide
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1. (a) $e \in G_s$: $e \cdot s = s$.
$g, h \in G_s \implies gh \in G_s$: $(gh) \cdot s = g \cdot (h \cdot s) = g \cdot s = s$.
$g \in G_s \implies g^{-1} \in G_s$: $g^{-1} \cdot s = g^{-1} \cdot (g \cdot s) = (g^{-1}g) \cdot s = e \cdot s = s$.

(b) No. Let G be the triangle group and let S be the set of vertices of the triangle. If $s \in S$ then G_s is not normal.

2. If $a \in G$, the coset of a is $\{a, ia, -a, -ia\}$. To prove $G/H \cong G$, consider the function $\phi : G \to G$, $\phi(z) = z^4$, and check that ϕ is a surjective homomorphism with kernel H.

3. We seek a sum of divisors of 55 that add up to 24, and we want this sum to have the smallest possible number of 1's. The solution is $11 + 11 + 1 + 1$, so the smallest possible number of fixed points is 2.

4. Answer: 33.

5. The Sylow p-subgroups of S_p are the cyclic groups $\langle a \rangle$, where a is a p-cycle. There are $(p-2)!$ such subgroups.

6. Answer: D_6.

7. $K = \mathbb{Q}[\sqrt{2}, i]$ and $G = \mathbb{Z}_2 \times \mathbb{Z}_2$ (the Klein 4-group). G has three proper non-trivial subgroups, all of index 2, and the three corresponding subfields of K are $\mathbb{Q}[\sqrt{2}]$, $\mathbb{Q}[i]$, and $\mathbb{Q}[i\sqrt{2}]$,