MATH 4010/6010 Exam 1
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C. McCrory

1. Define an isomorphism $\mathbb{Z}_{10}^{\times} \to \mathbb{Z}_4$, and check that it is an isomorphism.

2. Let G be a group, and let $a \in G$. Define a function $\phi : G \to G$ by

$$\phi(g) = aga^{-1}.$$

Prove that ϕ is an isomorphism.

3. Let G be the group of matrices

$$M = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

in $GL(2, \mathbb{R})$ such that $c = 0$. Let H be the subgroup of G consisting of matrices M with $c = 0$ and $a = 1$. Prove that H is a normal subgroup of G, and that the quotient group G/H is isomorphic to $\mathbb{R}^{\times}$.

4. How many elements of the symmetric group S_5 are conjugate to the element (123)? Explain your reasoning.

5. Prove: Let p be a prime and let m be a positive integer. If a group of order p^m acts on a set with n elements, and p does not divide n, then the action has a fixed point.