

MATH 4010/6010 Exam 2
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1. Give the definitions of the following terms:
 - (a) Sylow p -subgroup
 - (b) Galois group
 - (c) Galois extension

2. Find the number of different circular bracelets that can be made with 3 red beads, 2 white beads, and 1 blue bead. Show your reasoning.

3. Let G be an abelian group, and let H be a subgroup. Suppose that there is a homomorphism $\varphi : G \rightarrow H$ such that $\varphi(h) = h$ for all $h \in H$. Let $K = \ker \varphi$. Prove that $G \cong H \times K$.

4. Prove that if the group G has order 22 and G has only one element of order 2, then G is cyclic.

5. Find the Galois group of the field $K = \mathbb{Q}[\sqrt[6]{2}, i\sqrt{3}]$ over the rational numbers \mathbb{Q} . Show your reasoning.