

MATH 4010/6010 Final Exam
May 7, 2009, 8:00 – 11:00 am
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Briefly explain your reasoning in each problem.

1. Suppose the group G acts on the set S . If $s \in S$, let $G_s = \{g \in G : g \cdot s = s\}$.
 - (a) Prove that G_s is a subgroup of G .
 - (b) Is G_s a normal subgroup of G ? Give a proof or a counterexample.
2. Let $G = \mathbb{C}^\times$ be the multiplicative group of non-zero complex numbers. Let $H = \{1, i, -1, -i\}$ be the set of fourth roots of unity, which is a subgroup of G . Describe the cosets of H in G , and prove that G/H is isomorphic to G .
3. If a group G of order 55 acts on a set S with 24 elements, what is the smallest possible number of fixed points?
4. Find the number of different circular necklaces that can be made from four orange beads, two pink beads, and two blue beads. (All eight beads are used for each necklace.)
5. Let p be a prime. What are the Sylow p -subgroups of the symmetric group S_p ? How many are there?
6. Compute the Galois group of the field extension $\mathbb{Q}[\sqrt[6]{3}, i]$ of the rational numbers \mathbb{Q} .
7. Let $f(x) = x^4 - 4 \in \mathbb{Q}[x]$. Find the splitting field K of $f(x)$, the Galois group G of $f(x)$, the subgroups of G , and the corresponding subfields of K .