MATH 4250/6250 Exam 1 Retest
Monday, March 2
C. McCrory

1. Compute the tangent, normal, and binormal vectors of the curve

$$\vec{\alpha}(t) = (\cos 2t, \sin 2t, t^2)$$

at $t = \pi$.

2. (a) State the Frenet equations for a space curve.

(b) Prove that if $\vec{\alpha}$ is a regular space curve with constant torsion $\tau = 0$, then the image of $\vec{\alpha}$ lies in a plane. (Prove that if the binormal vector is constant, then $\vec{\alpha}$ lies in a plane.)

(c) Prove that if $\vec{\alpha}$ is a regular space curve with constant torsion $\tau = 0$ and constant curvature $\kappa > 0$, then the image of $\vec{\alpha}$ lies in a circle of radius $1/\kappa$. (What is the center of the circle?)

3. (a) Show that the function

$$\vec{x}(u, v) = (u, uv, e^v)$$

is differentiable, and that the partial derivatives $\vec{x}_u$ and $\vec{x}_v$ are independent. (You do not have to check that the domain of $\vec{x}$ is open or that $\vec{x}$ is injective.)

(b) Find an equation for the tangent plane to the surface $\vec{x}$ at the point $\vec{x}(2, 1)$

4. Compute the coordinate transformation $\varphi(s, t) = (u, v)$ for the coordinate patches

$$\vec{x}(s, t) = (s/t, st, s), \quad t > 0,$$

$$\vec{y}(u, v) = (u, v, \sqrt{uv}), \quad u > 0, \quad v > 0,$$

of the surface $S = \{(x, y, z) \mid xy = z^2\}$. Find the domain of φ, and verify that the derivative matrix $D\varphi$ is nonsingular. (You do not have to check that φ is bijective.)

Formulas for a non-unit speed curve $\vec{\alpha}(t)$. Prime denotes derivative with respect to t:

$\vec{T} = \vec{\alpha}'/|\alpha'|$

$\vec{B} = (\vec{\alpha}' \times \vec{\alpha}'')/|\vec{\alpha}' \times \vec{\alpha}''|$

$\vec{N} = \vec{B} \times \vec{T}$

$\kappa = |\vec{\alpha}' \times \vec{\alpha}''|/|\vec{\alpha}'|^3$

$\tau = (\vec{\alpha}' \times \vec{\alpha}'') \cdot \vec{\alpha}'''/|\vec{\alpha}' \times \vec{\alpha}''|^2 = [\vec{\alpha}', \vec{\alpha}'', \vec{\alpha}''']/|\vec{\alpha}' \times \vec{\alpha}''|^2$