MATH 4250/6250 Exam 2
Friday, April 10
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1. Define the following invariants of surfaces:

(a) shape operator

(b) principal curvatures

(c) Gaussian curvature

(d) mean curvature

2. Prove that if a surface M contains a line L through the point p on M, then $K(p) \leq 0$, where $K(p)$ is the Gaussian curvature of M at p. Fill in the details of the following proof outline:

(a) Let $\vec{u}$ be a unit tangent vector to M at p in the direction of the line L. Then, if S_p is the shape operator of M at p, we have $S_p(\vec{u}) \cdot \vec{u} = 0$.

(b) It follows that the normal curvature $\mathbf{k}(\vec{u}) = 0$.

(c) Euler's formula implies that the principal curvatures k_1 or k_2 of M at p have opposite signs or one of them is zero.

(d) Therefore $K(p) \leq 0$.

3. (a) Compute the Gaussian curvature of the surface

$$\vec{x}(u, v) = (u, v, u^2/2 + v^3/3).$$

(b) For which (u, v) is $K = 0$, $K > 0$, $K < 0$?

4. Compute the principal curvatures for every point of the torus

$$\vec{x}(u, v) = ((2 + \cos u) \cos v, (2 + \cos u) \sin v, \sin u).$$

Shortcut: Use that the meridians (u-parameter curves) and parallels (v-parameter curves) are lines of curvature.