MATH 4250/6250 Formulas

Lagrange:

$|\vec{v} \times \vec{w}|^2 = |\vec{v}|^2|\vec{w}|^2 - (\vec{v} \cdot \vec{w})^2$

Frenet:

$\vec{T}' = \kappa\vec{N}, \quad \vec{N}' = -\kappa\vec{T} + \tau\vec{B}, \quad \vec{B}' = -\tau\vec{N}$

non-unit speed Frenet:

$\vec{T}' = \kappa\nu\vec{N}, \quad \vec{N}' = -\kappa\nu\vec{T} + \tau\nu\vec{B}, \quad \vec{B}' = -\tau\nu\vec{N}$

non-unit speed curves:

$$\vec{T} = \frac{\vec{\alpha}'}{|\vec{\alpha}'|}, \quad \vec{B} = \frac{\vec{\alpha}' \times \vec{\alpha}''}{|\vec{\alpha}' \times \vec{\alpha}''|}, \quad \vec{N} = \vec{B} \times \vec{T}, \quad \kappa = \frac{|\vec{\alpha}' \times \vec{\alpha}''|}{|\vec{\alpha}'|^3}, \quad \tau = \frac{(\vec{\alpha}' \times \vec{\alpha}'') \cdot \vec{\alpha}'''}{|\vec{\alpha}' \times \vec{\alpha}''|^2}$$

surfaces:

$E = \vec{x}_u \cdot \vec{x}_u, \quad F = \vec{x}_u \cdot \vec{x}_v, \quad G = \vec{x}_v \cdot \vec{x}_v$

$l = S(\vec{x}_u) \cdot \vec{x}_u = \vec{U} \cdot \vec{x}_{uu}, \quad m = S(\vec{x}_u) \cdot \vec{x}_v = \vec{U} \cdot \vec{x}_{uv}, \quad n = S(\vec{x}_v) \cdot \vec{x}_v = \vec{U} \cdot \vec{x}_{vv}$

$$K = \frac{ln - m^2}{EG - F^2}$$

surfaces of revolution:

$$K = \frac{g'(g''h' - g'h'')}{h((g')^2 + (h')^2)^2}, \quad k_\mu = \frac{g''h' - g'h''}{((g')^2 + (h')^2)^{\frac{3}{2}}}, \quad k_\pi = \frac{g'}{h((g')^2 + (h')^2)^{\frac{1}{2}}}$$

shape operator:

$S(\vec{\alpha}') \cdot \vec{\alpha}' = \vec{\alpha}'' \cdot \vec{U}$

$k(\vec{u}) = (\cos\theta)^2 k_1 + (\sin\theta)^2 k_2$

geodesic curvature:

$\kappa(\vec{\alpha})\vec{N} = k(\vec{\alpha}')\vec{U} + \kappa_g(\vec{\alpha})\vec{U} \times \vec{T}$ (unit speed)

$\vec{\alpha}'' = \nu'\vec{T} + (S(\vec{\alpha}') \cdot \vec{\alpha}')\vec{U} + \nu^2\kappa_g(\vec{\alpha})\vec{U} \times \vec{T}$ (non-unit speed)

covariant derivative:

$\nabla^M_{\vec{v}}\vec{Z} = \nabla^{\mathbb{R}^3}_{\vec{v}}\vec{Z} - (\nabla^{\mathbb{R}^3}_{\vec{v}}\vec{Z} \cdot \vec{U})\vec{U}$