MATH 4250/6250 Problem Set 1
Due Friday, January 16

1. (a) Which of the following are regular curves?
(i) $\vec{\alpha}(\theta) = (\cos\theta, 1 - \cos\theta - \sin\theta, -\sin\theta)$
(ii) $\vec{\beta}(\theta) = (2\sin^2\theta, 2\sin^2\theta\tan\theta, 0)$
(iii) $\vec{\gamma}(\theta) = (\cos\theta, \cos^2\theta, \sin\theta)$

(b) Find the tangent line to each of the above curves at $\theta = \pi/4$.

2. (a) For the helix $\vec{\alpha}(t) = (a\cos t, a\sin t, bt)$, with $a > 0$ and $b > 0$, what is the equation of the tangent line at $t = t_0$?

(b) Show that the angle between $d\vec{\alpha}/dt$ and $\vec{u} = (0, 0, 1)$ and is a constant (independent of t).

3. Let $\vec{\alpha}(t) = (e^t\cos t, e^t\sin t, 0)$. Prove that the angle between the position vector and the tangent vector is constant.

4. Let $\vec{\alpha}(t)$ be a regular curve. Suppose there is a point $\vec{a} \in \mathbb{R}^3$ such that $\vec{\alpha}(t) - \vec{a}$ is orthogonal to $\vec{\alpha}'(t)$ for all t. Prove that $\vec{\alpha}(t)$ lies on a sphere. (*Hint:* What should be the center of the sphere?)

5. Oprea, Exercise 1.1.22 (equation of a catenary)

6. Find the arc length of $\vec{\alpha}(t) = (2\cosh 3t, -2\sinh 3t, 6t)$ for $0 \le t \le 5$.

7. Show that $\vec{\alpha}(s)$ is a unit speed curve, where
$$\vec{\alpha}(s) = \frac{1}{2}(s + \sqrt{s^2+1}, (s + \sqrt{s^2+1})^{-1}, \sqrt{2}\ln(s + \sqrt{s^2+1})).$$

8. Let $\vec{\alpha}(t)$ be a regular curve with $|d\vec{\alpha}/dt| = a$, where a is a fixed positive constant. Show that if s is arclength measured from some point, then $t = (s/a) + c$ for some constant c.

9. Oprea, Exercise 1.2.7 (involute of a curve)

10. Oprea, Exercise 1.2.8 (involute of a helix)