

MATH 4250/6250 Problem Set 2  
Due Friday, January 23

1. Show that each of the following curves is unit speed, and compute the Frenet frame  $(\vec{T}, \vec{N}, \vec{B})$ , the curvature  $\kappa$ , and the torsion  $\tau$ .

(a)  $\vec{\alpha}(s) = (\frac{1}{3}(1+s)^{\frac{3}{2}}, \frac{1}{3}(1-s)^{\frac{3}{2}}, \frac{1}{\sqrt{2}}s)$

(b)  $\vec{\beta}(s) = \frac{1}{2}(\arccos s - s\sqrt{1-s^2}, 1-s^2, 0)$

(c)  $\vec{\gamma}(s) = \frac{1}{\sqrt{5}}(\sqrt{1+s^2}, 2s, \ln(s + \sqrt{1+s^2}))$

2. Oprea, Exercise 1.3.22 (curve with all tangent lines through a point)

3. Oprea, Exercise 1.3.27 (curvature and torsion)

4. Oprea, Exercise 1.4.7 and 1.4.8 (hyperbolic helix)

5. For each of the following curves, compute the Frenet frame  $(\vec{T}, \vec{N}, \vec{B})$ , the curvature  $\kappa$ , and the torsion  $\tau$ .

(a)  $\vec{\alpha}(t) = (t, t^2, t^3), t > 0.$

(b)  $\vec{\beta}(t) = (t - \cos t, \sin t, t)$