MATH 4250/6250 Problem Set 3, Part A
Due Friday, January 30

1. Oprea, Exercise 1.4.16 (evolute of an astroid – compare the Maple exercise 1.7.6)

2. Oprea, Exercises 1.5.3, 1.5.7, 1.5.8 (helix geometry)

3. If $\vec{\alpha}$ is a regular curve, the normal plane at $\vec{\alpha}(t)$ is the plane through $\vec{\alpha}(t)$ perpendicular to the tangent vector $\vec{T}$. Suppose there is a point $\vec{p}$ that belongs to every normal plane of $\vec{\alpha}$. Prove that $\vec{\alpha}$ lies on a sphere.

4. Let $\vec{\alpha}(s)$ be a regular curve that lies on a sphere with center $\vec{p}$ and radius r. Show that $\kappa \neq 0$, and that if $\tau \neq 0$ then

$$\vec{\alpha} - \vec{p} = -\rho\vec{N} - \rho'\sigma\vec{B},$$

where $\rho = 1/\kappa$ and $\sigma = 1/\tau$. Hence $r^2 = \rho^2 + (\rho'\sigma)^2$.

Hint: Assume $\vec{\alpha}$ is parametrized by arclength. For the second part, write $\vec{\alpha} - \vec{p} = a\vec{T} + b\vec{N} + c\vec{B}$, and compute the functions a, b, c.

5. If $\vec{\alpha}$ is a regular curve with non-vanishing curvature, the normal line at $\vec{\alpha}(t)$ is the line through $\vec{\alpha}(t)$ spanned by the principal normal $\vec{N}$. Suppose that the curves $\vec{\alpha}$ and $\vec{\beta}$ have the property that for every t the normal line at $\vec{\alpha}(t)$ equals the normal line at $\vec{\beta}(t)$.

(a) Prove that the distance between corresponding points $\vec{\alpha}(t)$ and $\vec{\beta}(t)$ is constant.

(b) Prove that the angle between the tangent vectors at corresponding points is constant.

Note: One may assume that one of the curves is parametrized by arclength, but not both.

Part B of the problem set is a lab assignment.