MATH 4250/6250 Problem Set 4
Due Friday, February 6

Part A. Check that the following functions are surface coordinate patches. (See Oprea, page 68, for the definition of a coordinate patch.)

(a) $\vec{x}(u,v) = (u+v, u-v, uv)$, $-\infty < u < \infty$, $-\infty < v < \infty$.

(b) $\vec{x}(u,v) = (g(u), h(u)\cos v, h(u)\sin v)$, where $\vec{\alpha}(u) = (g(u), h(u), 0)$ is a regular plane curve with $h(u) > 0$ for all u, $a < u < b$, $-\pi < v < \pi$ (Oprea, Example 2.1.10, page 72).

(c) $\vec{x}(u,v) = ((R + r\cos u)\cos v, (R + r\cos u)\sin v, r\sin u)$, with $R > r$, $-\pi < u < \pi$, $-\pi < v < \pi$ (Oprea, Exercise 2.1.13). (This is the previous example with $g(u) = r\sin u$, $h(u) = R + r\cos u$, and the coordinates in $\mathbb{R}^3$ permuted, so that the axis of rotation is the z-axis rather than the x-axis.)

(d) $\vec{x}(u,v) = (\cos u, \sin u, 0) + v(\sin\frac{1}{2}u\cos u, \sin\frac{1}{2}u\sin u, \cos\frac{1}{2}u)$, $-\pi < u < \pi$, $-\frac{1}{2} < v < \frac{1}{2}$.

Part B. Graph the above surfaces using our graphics courseware. Go to the Differential Geometry Lab link on the course web page.