MATH 3000
MIDTERM 1

Name: ______________________________

Problem 1: (a) (15 points) Use Gaussian elimination to give the general solution of the following system of equations in standard (i.e. parametric) form:

$$\begin{aligned} 2x_1 \qquad\quad - 4x_3 + \ \ x_4 &= \ \ 2 \\ 3x_1 + 2x_2 + 3x_3 \qquad\quad &= -1 \\ 4x_2 + 18x_3 - 3x_4 &= -8 \end{aligned}$$

(b) (10 points) True or false:

$$\text{Span}\left(\begin{bmatrix}2\\3\\0\end{bmatrix}, \begin{bmatrix}0\\2\\4\end{bmatrix}, \begin{bmatrix}-4\\3\\18\end{bmatrix}, \begin{bmatrix}1\\0\\-3\end{bmatrix}\right) = \mathbb{R}^3.$$

Justify your answer.

Date: June 24, 2007.

Problem 2: (25 points) Consider a triangle ABC inscribed in a circle. Suppose $\overline{AC}$ is a diameter of the circle. Show that $\angle ABC$ is a right angle.

Problem 3: Let $P \subset \mathbb{R}^3$ be the plane containing the points $(1,1,1), (-1,2,3)$, and $(0,4,0)$.

(a) (5 points) Write down a parametric equation for P.

(b) (20 points) Write down a Cartesian equation for P.

Problem 4: (a) (15 points) Let α be a real number. For which values of α will the matrix

$$A_\alpha = \begin{bmatrix} 1 & 1 & \alpha \\ \alpha & 2 & \alpha \\ \alpha & \alpha & 1 \end{bmatrix}$$

be singular?

(b) (10 points) For each of the values of α you found in part (a), give constraint equations on $\mathbf{b}$ for the equation $A_\alpha \mathbf{x} = \mathbf{b}$ to be consistent. (What happens when α is not one of the values you found in part (a)?)