

## MATH 3000, QUIZ 1

Consider the plane in  $\mathbb{R}^3$  given by the parametric (explicit) equation

$$\mathbf{x} = (1, 0, 1) + s(2, 1, 0) + t(-1, 0, 1), \quad s, t \in \mathbb{R}.$$

Find a Cartesian (implicit) equation for this plane. (Recall: a Cartesian equation for a plane in  $\mathbb{R}^3$  is an equation of the form  $a_1x_1 + a_2x_2 + a_3x_3 = c$ , for some real numbers  $a_1, a_2, a_3, c$ .) Of course, show your work and explain what you're doing! Use whatever method you want.

*Solution:* The normal vector  $\mathbf{a} = (a_1, a_2, a_3)$  must be perpendicular to  $(2, 1, 0)$  and  $(-1, 0, 1)$ , so we get the equations

$$2a_1 + a_2 = 0$$

$$-a_1 + a_3 = 0$$

from which we get  $a_1 = a_3$  and  $a_2 = -2a_1$ . So  $\mathbf{a} = (a_1, -2a_1, a_1)$ , and we can choose the normal vector however we want, so let's just take  $a_1 = 1$ . Then  $\mathbf{a} = (1, -2, 1)$ , and then the equation of the plane is

$$(1, -2, 1) \cdot (\mathbf{x} - (1, 0, 1)) = 0,$$

which becomes

$$x_1 - 2x_2 + x_3 = 2.$$