MATH 2500 - Swinarski
Midterm 1
February 11, 2009

No calculators, computers, or other electronic devices are permitted.
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No books, notes, or formula sheets are permitted.
$\sim$
Make sure to show all work.
$\sim$
You have 50 minutes to complete this exam.

Name : _____

Good luck!

Question	Points	Score
1	25	
2	20	
3	15	
4	20	
5	20	
Total:	100	

1. Let A, B, and C be the following points:

$$\begin{aligned} A &= (1, 2, 3), \\ B &= (-1, 0, 1), \\ C &= (2, 2, 4). \end{aligned}$$

(a) (15 points) Find an equation for the plane containing A, B, and C.

(b) (5 points) What is the area of the triangle ABC?

(c) (5 points) What is the angle at vertex B (that is, made by AB and BC)?

2. (a) (10 points) Graph the parametrized curve $\vec{r}(t) = (t, t^3, t), \quad -\infty < t < \infty$. (Hint: graph some projections to the coordinate planes.)

(b) (10 points) Find an equation for the tangent line of this curve through the point $(2, 8, 2)$.

3. (15 points) Find the arclength of the parametrized curve

$$\vec{r}(t) = (t, \ln|\sec t|, 1), \quad 0 \le t \le 1.$$

Hint: the following formulas may help.

$$\int \sec u \, du = \ln|\sec u + \tan u| + C, \qquad \int \tan u \, du = \ln|\sec u| + C$$

4. (20 points) A water balloon is launched from a slingshot from the edge of the roof of a dorm which is adjacent to a parking lot. The roof is 25m above the ground and flat. The slingshot is positioned so that the balloon is launched at an angle of 60 degrees relative to the roof, and its initial speed is 23m/s. How far away from the dorm will the balloon hit the ground? Use the approximations $g \approx 10\text{m/s}$, and $23\sin(60^\circ) \approx 20$. (Hint: begin by drawing a picture.)

5. Frenet frames are a topic we did not cover in class. But, every step of the calculation is something you know how to do.

   We will compute the Frenet frame for the twisted cubic when $t = 1$. In the interests of time, I have done some of the steps for you. Follow the instructions below to find three vectors called $\vec{T}$, $\vec{N}$, and $\vec{B}$. This completes the calculation of the Frenet frame.

   Recall that the twisted cubic is given by the equations

$$\vec{r}(t) = (t, t^2, t^3).$$

   Then it is easy to compute that

$$\begin{aligned} \vec{r}\,'(t) &= (1, 2t, 3t^2) \\ \vec{r}\,''(t) &= (0, 2, 6t) \end{aligned}$$

   and if we plug in $t = 1$, then we get:

$$\begin{aligned} P = \vec{r}(1) &= (1, 1, 1) \\ \vec{r}\,'(1) &= (1, 2, 3) \\ \vec{r}\,''(1) &= (0, 2, 6) \end{aligned}$$

   (a) (2 points) Find a vector of length 1 in the direction of $\vec{r}\,'(1)$. This is $\vec{T}$.

   (b) (3 points) Compute $\text{proj}_{\vec{r}\,'(1)}\vec{r}\,''(1)$.

(c) (2 points) Use your answer from part (b) to compute the following vector $\vec{n}$:

$$\vec{n} = \vec{r}''(1) - \text{proj}_{\vec{r}'(1)}\vec{r}''(1)$$

(d) (2 points) Check that $\vec{r}'(1)$ and $\vec{n}$ are orthogonal. (If you did part (c) correctly, they will be.)

(e) (2 points) Find a vector of length 1 in the same direction as $\vec{n}$. Call this $\vec{N}$.

(f) (3 points) Compute $\vec{b} = \vec{r}'(1) \times \vec{r}''(1)$.

(g) (2 points) Find a vector of length 1 in the direction of $\vec{b}$. Call this $\vec{B}$.

(h) (2 points) Is $\vec{B}$ orthogonal to $\vec{T}$? Is $\vec{B}$ orthogonal to $\vec{N}$?

(i) (2 points) Sketch the point $P = \vec{r}(1)$. (You don't need to sketch the whole curve, just this point.) Sketch $\vec{T}$, $\vec{N}$, and $\vec{B}$ with their tail ends placed at P. Congratulations! This is a Frenet frame for $\vec{r}(t)$ at P.