MATH 2500 - Swinarski
Midterm 2
March 20, 2009

No calculators, computers, or other electronic devices are permitted.
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No books, notes, or formula sheets are permitted.
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Make sure to show all work.
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You have 50 minutes to complete this exam.

Name : ______________________________

Good luck!

Question	Points	Score
1	10	
2	15	
3	10	
4	20	
5	25	
6	20	
Total:	100	

1. (10 points) Let

$$w = z - \sin(xy),$$

and suppose that x, y, and z are in turn functions of t as given below:

$$\begin{aligned} x &= t \\ y &= \ln t \\ z &= e^{t-1} \end{aligned}$$

(a) Use the Chain Rule to compute a formula for dw/dt.

(b) What is dw/dt when $t = 1$?

2. (15 points) Let S be the surface defined by the equation

$$F(x, y, z) = \ln(x^2 + y^2) - z.$$

Find an equation for the tangent plane of S through the point $(0, 1, 0)$.

3. Suppose f is a function of x and y, and the gradient of f at the point $P = (2, 0)$ is $\nabla f(2, 0) = (4, -8)$.

   (a) (5 points) What direction $\mathbf{u}$ gives the largest directional derivative $D_{\mathbf{u}} f(2, 0)$? (Remember, your answer should be a unit vector.)

   (b) (5 points) When $\mathbf{u}$ is your answer from part (a), what is $D_{\mathbf{u}} f(2, 0)$?

4. (20 points) *The second derivative test.* Let

$$f(x, y) = x^4 + y^4 + 4xy.$$

Find all the critical points of f. For each critical point, use the second derivative test to determine whether the critical point is a local maximum, a local minimum, or a saddle point, or that the test is inconclusive.

5. (25 points) Let

$$f(x, y) = x^2 - \frac{1}{2}y + \frac{1}{2}y^2$$

on the domain D, where D is the upper half of the unit disk. D is given by the inequalities $y \geq 0$ and $x^2 + y^2 \leq 1$. Find the absolute minima and maxima of f on D.

6. (20 points) *Lagrange multipliers.* Use Lagrange multipliers to find the maxima and minima of the function

$$f(x, y) = x^2 y$$

subject to the constraint

$$x^2 + 2y^2 = 4.$$

Note: there may be other valid methods for solving this problem, but to get full credit, you must use Lagrange multipliers.