Study guide for Quiz 3 on Friday, April 23
MATH 4790/6900 - Swinarski

Here are some practice questions or study hints for Quiz 3. Remember that Quiz 1+ Quiz 2 +Quiz 3 = Test 3. Quiz 3 will only be 25 minutes long, so there can be at most 3 or 4 questions. I won't ask you for any proofs (that was Quiz 2). I will give you the Black-Scholes PDE and formulas from Quiz 1. I'll also give you the formulas for the price of a European put, the price of a digital call, and Green's function for the Black-Scholes PDE. However, I won't give you Itô's Lemma, the formulas for Brownian motion, or the Greeks.

As usual, you should make sure you understand all the homework problems from Homeworks 8-11. Here are some additional questions you should be prepared for.

1. Brownian motion

- Be prepared to define Brownian motion. We had two definitions: Brownian motion can be defined as the limit of scaled symmetric random walks. Alternatively, it can be defined as a process which has independent normally distributed increments, starts at 0, and has continuous paths.
- I won't ask you to graph $W^{(n)}(t)$ as I did on Homework 8. However, I might give you a sequence of coin flips, e.g. $HHTTHTTHHHH...$ and ask you to identify $W^{(n)}(t)$ from a lineup of graphs. Look for the graph which has the correct horizontal and vertical scaling.
- Why is arithmetic Brownian motion a poor choice for modeling stock prices? Why is geometric Brownian motion a better choice for modeling stock prices?
- Given pictures a Brownian motion, determine whether it is arithmetic or geometric Brownian motion. (What to look for: are the sizes of the zigzags independent of the height (ABM) or proportional to the height (GBM)?)

2. Black-Scholes

- Given a stock which pays no dividends, an exchange rate, or a stock which pays dividends, and all the necessary data (S, K, r, q, σ, $T-t$), compute the price of a European call, a European put, or a digital call.
- Given a portfolio of options (e.g. a bull spread), compute its value. (Price each option in the portfolio.)
- What is the definition of volatility? What is implied volatility? How does it compare to historical volatility? (We don't know how to compute historical volatility yet—that's our next computer lab—but I want you to know where it fits in the theory.)
- Be prepared to use Itô's Lemma (cf. Homework 9 questions 1 and 2).
- Be prepared to give a one-paragraph summary of how we found the Black-Scholes price of a European call option in class lectures. Here's my answer: *We found an explicit coordinate transformation under which the Black-Scholes PDE became the heat equation. We also transformed the boundary condition (payoff diagram for a European call) to the heat equation variables. Then we found the solution using Green's function for the heat equation, and changed coordinates back to the original variables, yielding the formula* $f(S,t) = SN(d_1) - Ke^{-r(T-t)}N(d_2)$ *for the price of a European call.*
- Given a new kind of derivative contract defined by its payoff diagram, find a formula for its price. (Answer: put the payoff diagram as $h(z)$ inside the integral in Green's function, and leave your answer as an integral.)
- Be prepared to compute partial derivatives of f like we did in the homework, or of related functions (for instance, partial derivatives of the European put price function, or second derivatives of f...).

3. And don't forget to look over Homeworks 8-11...