Name:__________________________

MATH 2200, Fall 2009
Exam 2 — PRACTICE

Please hand only only clearly written work, not scratch paper. Clearly mark your final answers for each problem. Partial credit will only be given on problems for which your work is clearly shown.

The only allowable materials for this exam are paper, pens and pencils. No notes, textbooks or calculators will be allowed.

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(1) Consider the function

$$f(x) = 3x^2 + 2x - 1 \quad \text{on the interval } [1, 3]$$

(a) Find all critical points of $f(x)$ on this interval.

Critical points:

(b) Find the x-values at which the minimum and maximum of the function are achieved.

Minimum at $x =$

Maximim at $x =$

Please show your work clearly below.

(2) Consider the function

$$f(x) = x^2 - x \quad \text{on the interval } [-1, 2)$$

(a) Find all critical points of $f(x)$ on this interval (or write 'none').

Critical points: [ ]

(b) Construct sign chart for the derivative of $f(x)$ in the space below, illustrating where the function $f(x)$ is increasing and decreasing.

[ ]

Show your work below:

(c) Find the minimum value obtained by $f(x)$ on the given interval:

minimum value : [ ]

(3) Consider the function

$$f(x) = 3x^3 + x \qquad \text{on the interval } (-1, 1/3)$$

(a) Find all critical points of $f(x)$ on this interval (or write 'none').

Critical points: [ ]

(b) Construct sign chart for the derivative of $f(x)$ in the space below, illustrating where the function $f(x)$ is increasing and decreasing.

[ ]

Show your work below:

(c) Find the maximum value obtained by $f(x)$ on the given interval:

maximum value : [ ]

(4) A company would like to construct a cylindrical can with an open top at a fixed cost of $\$200\pi$. If the cost of the materials for the sides of the can is \$2 per square foot, and the cost of materials for the bottom of the can is \$5 per square foot, what are the dimensions of the can (radius, height) which will give the largest possible volume?

Potentially useful formulas:

Volume of a cylinder $= \pi r^2 h$ Surface area around a cylinder $= 2\pi r h$

Area of a circle $= \pi r^2$

(a) What is the quantity which you are trying to maximize?

Write an equation for the quantity to be maximized, as a function of only one variable, and state what the meaning of this variable is (i.e. radius, diameter, height, etc.).

Equation :

Use the space below to show your work.

(b) What is the interval on which this function is defined?

You may justify your answer below (only for partial credit if above answer if incorrect).

(c) Find all critical points of this function on the interval you have found.

Use the space below to show your work.

(d) Find the dimensions of the can (radius, height) which achieve the maximum volume.

Dimensions:

Use the space below to show your work.

(5) A company would like to construct a cylindrical can with an open top with a volume of 4π cubic feet. If the cost of the materials for the sides of the can is \$2 per square foot, and the cost of materials for the bottom of the can is \$5 per square foot, what are the dimensions of the can (radius, height) which will minimize the cost of the materials?

Potentially useful formulas:

Volume of a cylinder $= \pi r^2 h$ Surface area around a cylinder $= 2\pi r h$

Area of a circle $= \pi r^2$

(a) What is the quantity which you are trying to minimize?

Write an equation for the quantity to be minimized, as a function of only one variable, and state what the meaning of this variable is (i.e. radius, diameter, height, etc.).

Equation :

Use the space below to show your work.

(b) What is the interval on which this function is defined?

You may justify your answer below (only for partial credit if above answer if incorrect).

(c) Find all critical points of this function on the interval you have found.

Use the space below to show your work.

(d) Find the dimensions of the can (radius, height) which achieve the minimum cost of materials.

Dimensions:

Use the space below to show your work.

(6) A farmer would like to set up a rectangular enclosure of fencing in order to enclose a fixed area of 1000 square feet. If three of the sides of the enclosure cost \$2 per linear foot of fencing, and the fourth side costs \$7 per linear foot, find the dimensions of the enclosure which minimize the total cost.

(a) What is the quantity which you are trying to minimize?

Write an equation for the quantity to be minimized, as a function of only one variable, and state what the meaning of this variable is (i.e. radius, diameter, height, etc.).

Equation :

Use the space below to show your work.

(b) What is the interval on which this function is defined?

You may justify your answer below (only for partial credit if above answer if incorrect).

(c) Find all critical points of this function on the interval you have found.

Use the space below to show your work.

(d) Find the dimensions of the fence (length, width, height) which minimize the total cost.

Dimensions:

Use the space below to show your work.

(7) A company would like to construct a box with a square base in such as way as to enclose the maximum possible volume at a fixed cost of \$100. The sides and the bottom of this box will each cost \$2 per square foot, and the top will cost \$7 per square foot. Find the dimensions of the box which will maximize the volume at this cost.

(a) What is the quantity which you are trying to maximize?

Write an equation for the quantity to be maximized, as a function of only one variable, and state what the meaning of this variable is (i.e. radius, diameter, height, etc.).

Equation :

Use the space below to show your work.

(b) What is the interval on which this function is defined?

You may justify your answer below (only for partial credit if above answer if incorrect).

(c) Find all critical points of this function on the interval you have found.

Use the space below to show your work.

(d) Find the dimensions of the box (length, width, height) which achieve the maximum volume at the price of $100.

Dimensions:

Use the space below to show your work.

(8) Let $f(x) = x^5$.

(a) Find an equation for the tangent line to the graph of $f(x)$ at $x = 2$. Equation for tangent line : y = [ ]

Use the space below to show your work.

(b) Use this to give an approximation to $(2.1)^5$ and $(1.99)^5$. Your answer should be in the form of a decimal number.

$(2.1)^5 \approx$ [ ]

$(1.99)^5 \approx$ [ ]

(9) Let $f(x) = \sqrt{x}$.

(a) Find an equation for the tangent line to the graph of $f(x)$ at $x = 16$. Equation for tangent line : y =

Use the space below to show your work.

(b) Use this to give an approximation to $\sqrt{17}$ and $\sqrt{15}$. Your answer should be in the form of a whole number or a ratio of whole numbers.

$\sqrt{17} \approx$

$\sqrt{15} \approx$