MATH 2200, Fall 2009
Practice Final

(1) Compute the following derivatives:

(a)

$$\frac{d}{dx}\left(x^4(3-2x^3-x)x^3\right)$$

(b)

$$\frac{d}{dy}\left(\frac{e^y-\sin(y)}{y^2-\ln y}\right)$$

(c)

$$\frac{d}{dt}\left(\sin(\sin(e^{\cos(t)}))\right)$$

(d)

$$\frac{d}{ds}\left(\frac{\ln s-s}{s\ln s-s}\right)$$

(2) Compute the following:

(a) Suppose $f(x) = \sin(x)\cos(x)$. What is $f''(x)$?

(b) Suppose $y = f(x)$ is a function such that

$$3y^2x + 2xy - sin(x + y) = 13$$

Find $\frac{d}{dx}$ in terms of x and y.

(3) Consider the function

$$f(x) = e^x$$

Use linear approximation to estimate the value of $f(1.1)$.

(4) Suppose $y = f(x)$ is a function which satisfies the equation

$$x^2 - 4y^3 = 5$$

and such that $f(3) = 1$.

(a) Find the slope of the tangent line to the graph of $f(x)$ at the point $(3, 1)$.

(b) Use linear approximation to estimate the value of $f\left(3 + \frac{2}{17}\right)$.

(5) Calculate the following indefinite integrals:

(a)

$$\int \left(2x^4 - 2x + \frac{1}{x^2}\right) dx$$

(b)

$$\int x \sin(x^2 - 3) dx$$

(c)

$$\int \frac{e^{1/x}}{x^2} dx$$

(d)

$$\int \frac{1}{5} e^{4t-3} dt$$

(e)

$$\int (\cos(x) - 3\sin(2x))\, dx$$

(6) Suppose y is a function of x such that

$$\frac{dy}{dx} = 4x^2 - 2$$

and $y(1) = 2$. Find y.

(7) Suppose y is a function of x such that

$$y\frac{dy}{dx} = 4x^2 - 2$$

and $y(1) = 2$. Find y.

(8) Suppose y is a function of x such that

$$(y+3)\frac{dy}{dx} = y\sin(2x)$$

and $y(0) = 3$. Find y.

(9) Suppose y is a function of x such that

$$\frac{dy}{dx} = 4(y-3)^3 - \sin(x+y-7) + 3$$

and $y(4) = 3$.

(a) Find the slope of the tangent line to the graph of the function y at the point $(4, 3)$.

(b) Use linear approximation to estimate the value of $y(4 + \frac{1}{10})$.

(10) Consider the function

$$f(x) = \ln x + \frac{7}{x} - 3 \quad \text{on the interval } [1, \infty)$$

(a) Find all critical points of $f(x)$ on this interval (or write 'none').

Critical points:

(b) Construct sign chart for the derivative of $f(x)$ in the space below, illustrating where the function $f(x)$ is increasing and decreasing.

Show your work below:

(c) Find the minimum value obtained by $f(x)$ on the given interval:

minimum value :

(11) A farmer would like to set up a rectangular enclosure of fencing at a fixed cost of \$100. If three of the sides of the enclosure cost \$2 per linear foot of fencing, and the fourth side costs \$5 per linear foot, find the dimensions of the enclosure which maximize the total area at this cost.

(a) What is the quantity which you are trying to maximize? [ ]

Write an equation for the quantity to be maximized, as a function of only one variable, and state what the meaning of this variable is (i.e. radius, diameter, height, etc.).

Equation : [ ]

Use the space below to show your work.

(b) What is the interval on which this function is defined? [ ]

You may justify your answer below (only for partial credit if above answer if incorrect).

(c) Find all critical points of this function on the interval you have found.

Use the space below to show your work.

(d) Find the dimensions of the fence (length, width) which maximize the total area.

Dimensions:

Use the space below to show your work.

(12) Suppose the demand of a given commodity is related to its price by the equation

$$D \ln(D/10) + 1 = \frac{1}{p^2}$$

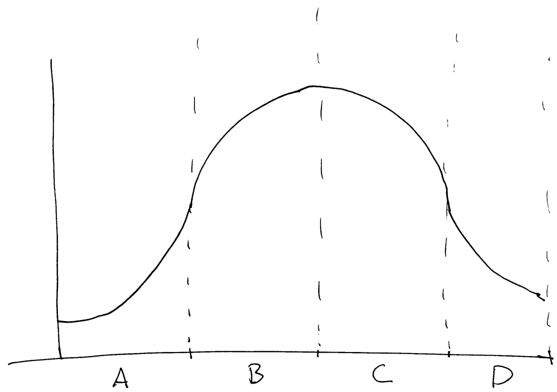
where D is the demand and p is the price, and recall that the revenue may be expressed as

$$R = Dp.$$

(a) Find the rate of change of the revenue with respect to price when $p = 10$ and $D = 10$

(b) Use linear approximation to estimate the revenue when the price is 11.

(13) Consider the following graph of a function $f(x)$:



In the spaces below, write either a '+' or a '−' to describe the intervals on which the first and second derivatives of $f(x)$ are positive or negative.

	A	B	C	D
f'(x)				
f''(x)				