Anti-Power Rule

$$\int x^n dx = \frac{1}{n+1} x^{n+1} + C$$

Anti-Chain Rule : u-substitution

Chain Rule gives us:

Suppose $\int f(u)\,du = F(u) + C$

(i.e. we know $\frac{d}{du} F(u) = f(u)$)

then $\int f(g(x))\,g'(x)\,dx = F(g(x)) + C$

since $\frac{d}{dx} F(g(x)) = F'(g(x)) \cdot g'(x)$

$= f(g(x))\,g'(x)$

Pattern to match:

$$\int \underbrace{f(g(x))}\,\underbrace{g'(x)\,dx}$$

$\leftarrow$ substituting u for $g(x)$
and du for $g'(x)\,dx$.

$$\int f(u)\,du$$

<u>example:</u>

$$\int e^{x^2}\cdot 2x\,dx$$

$u = x^2 \qquad du = \frac{d}{dx}(x^2)\cdot dx$

$\qquad\qquad\qquad = 2x\,dx$

$$= \int e^u\,du = e^u + C = e^{x^2} + C$$

Find a function e^{x^2} whose der. is

$$e^{x^2}\cdot 2x$$

- $\int \sin(2x)\cdot 2\,dx = \int \sin u \cdot du$

$u = 2x$ $\qquad = -\cos u + C$

$du = \frac{d}{dx}(2x)\cdot dx$ $\qquad = -\cos 2x + C$

$du = 2\cdot dx$

- $\int \underbrace{\frac{1}{x}(\overbrace{1+\ln x}^{u})^7\,dx}_{du}$

$u = 1+\ln x$ $\qquad du = \frac{d}{dx}(1+\ln x)\cdot dx$

$= \frac{1}{x}dx$

$= \int u^7\,du = \frac{1}{7+1}u^{7+1} + C = \frac{1}{8}u^8 + C$

$= \frac{1}{8}(1+\ln x)^8 + C.$

$$\int (\sin 3x) \cdot 4\,dx = \int u \cdot$$

$u = \sin 3x$

$du = \frac{d}{dx}(\sin 3x)\,dx = \cos 3x \cdot 3\,dx$

$u = 3x$

$du = \frac{d}{dx} 3x\,dx = 3\,dx$

$$= \int \sin u$$

$$= \int (\sin 3x) \frac{3}{3} 4\,dx = \int \frac{4}{3} (\sin 3x)\, 3\,dx$$

$$= \int \frac{4}{3} \sin u\,du = \frac{4}{3} \int \sin u\,du$$

$$= \frac{4}{3}(-\cos u) + C$$

$$= -\frac{4}{3} \cos 3x + C$$

$\bullet \int (2x+3)^7 \, dx = \frac{1}{2}\int (2x+3)^7 \cdot 2 \, dx$

$u = 2x+3$

$du = 2 \cdot dx$

$= \frac{1}{2}\int u^7 du = \frac{1}{2} \cdot \frac{1}{8} u^8 + C$

$= \frac{1}{16}(2x+3)^8 + C.$

Practice:

1. $\int e^{\sin x} \cos x \, dx$

2. $\int \frac{\ln(x+3)}{x+3} \, dx$

3. $\int e^{3x+1} dx$

4. $\int e^{3x^2+2} x \, dx$

5. $\int e^{-x} dx$

1. $\int e^{\sin x} \cos x \, dx$

$u = \sin x$

$du = \frac{d}{dx} \sin x \, dx$

$= \cos x \, dx$

$\frac{du}{dx} = \frac{d}{dx} \sin x = \cos x$

$\frac{du}{dx} = \cos x$

$du = \cos x \, dx$

$$\int e^u \, du = e^u + C = \underline{e^{\sin x} + C}.$$

$$\left[\int x \, dx = \frac{x^2}{2} + C \right]$$

2. $\int \frac{\ln(x+3)\,dx}{x+3} = \int \ln(x+3) \cdot \frac{1}{x+3} \, dx$

$u = \ln(x+3)$

$du = \frac{1}{x+3} \frac{d}{dx}(x+3) \, dx$

$= \frac{1}{x+3} \cdot dx$

$$= \int u \, du = \frac{u^2}{2} + C$$

$$= \frac{(\ln(x+3))^2}{2} + C$$

$$\left[\int u\,du = \int u^1\,du = \frac{1}{1+1}u^{1+1} + C \right]$$

3. $\int e^{3x+1}\,dx = \int e^u \ldots$

$u = 3x+1$

$du = 3\,dx \qquad \frac{1}{3}du = dx$

$$\int e^{3x+1}\cdot 1\cdot dx = \int e^{3x+1}\frac{3}{3}dx =$$

$$= \frac{1}{3}\int e^{3x+1}\cdot 3\,dx$$

$$= \frac{1}{3}\int e^u\,du$$

$$= \frac{1}{3}e^u + C$$

$$= \frac{1}{3}e^{3x+1} + C$$

$$\int \sin 2x\,dx = \int \sin u \,\tfrac{1}{2}\,du$$

$u = 2x$

$$= \frac{1}{2}\int \sin u\,du$$

$du = 2\,dx$

$\frac{1}{2}du = dx$

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4. $$\int e^{3x^2+2}\,\frac{6}{6}x\,dx = \int e^{u}\,\frac{1}{6}\,du$$

$u = 3x^2+2$

$du = 6x\,dx$

$$= \frac{1}{6}\int e^{u}\,du$$

$$= \frac{1}{6}e^{u} + C$$

$$= \frac{1}{6}e^{3x^2+2} + C$$

5. $\displaystyle\int e^{-x}\frac{(-1)}{(-1)}dx = \int e^{u}\frac{\cancel{1}}{-1}du = -\int e^{u}du$

$u = -x$

$du = -dx$

$-du = dx$

$$\int e^{-x}dx = \int e^{u}(-du) = -\int e^{u}du$$

$$= -e^{u} + C$$

$$= -e^{-x} + C.$$

$$\int \sin x \cos x \, dx$$

$u = \sin x$

$du = \cos x \, dx$

$\int u \, du$ ← find an anti-derivative for u

$\frac{u^2}{2}$

$\Rightarrow \frac{u^2}{2} + C$

$\Rightarrow \boxed{\frac{(\sin x)^2}{2} + C}$

$\int x^2 dx = \frac{1}{3} x^3 + C$

$\int \sin u \, du = -\cos u + C$

$$\int \tan x \, dx = \int \frac{\sin x}{\cos x} dx = \int \frac{1}{u} \cdot (-du)$$

$u = \cos x$

$du = -\sin x \, dx$

$-du = \sin x \, dx$

$$= -\int \frac{1}{u} du = -\ln u + C = -\ln(\cos x) + C$$

$$= \ln(\cos x)^{-1} + C = \ln(\sec x) + C$$