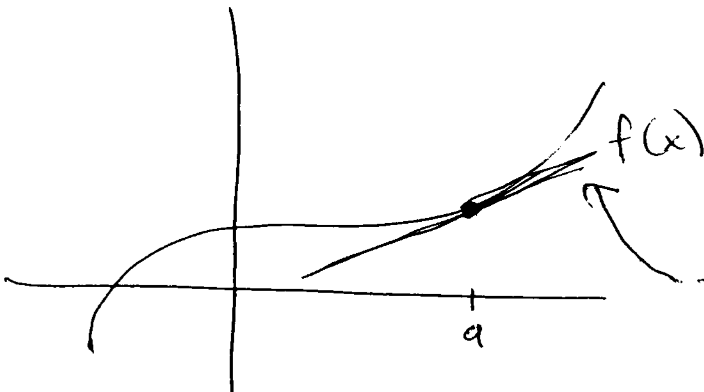
Last time

start w/ some function $f(x)$

(e.g. quadratic functions)

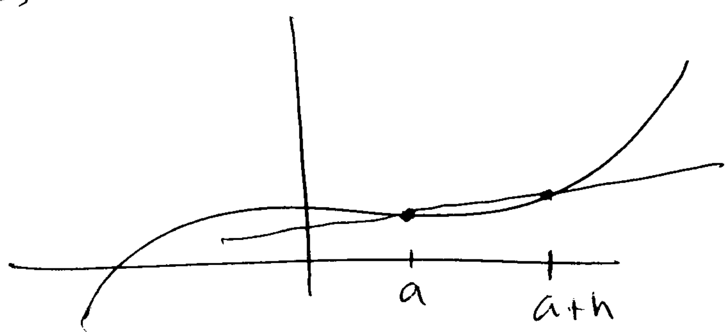
calculated rate of change of $f(x)$ at some value $x=a$



rate of change of $f(x)$ at a is slope of tangent line

$m(a)$

To find $m(a)$ looked at slopes of secant lines



slope of secant line

$$\frac{f(a+h)-f(a)}{(a+h)-a}$$

slope of secant line

$$\lim_{h \to 0} \frac{f(a+h) - f(a)}{h} = m(a)$$

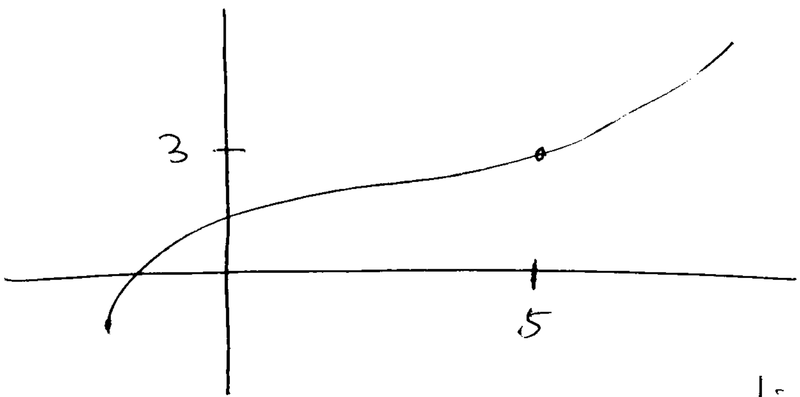
Limits

Def $\lim_{x \to a} f(x) = L$

Sentence means as x gets close to a $f(x)$ gets close to L.

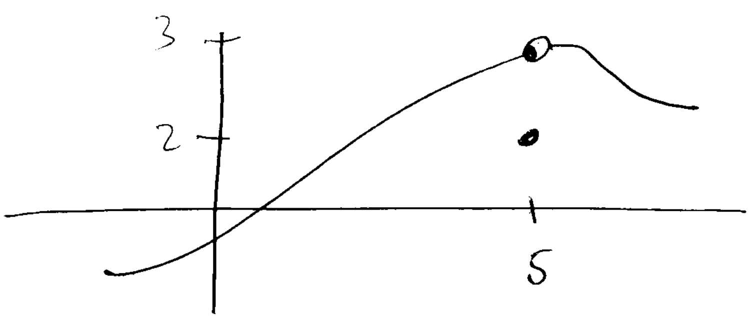
i.e. given any positive number ε (no matter how small), ~~then~~ there exists some (maybe very small) number δ such that whenever $|x-a| < \delta$ ~~then~~ we have $|f(x) - L| < \varepsilon$

How to deal w/ limits



$f(x)$

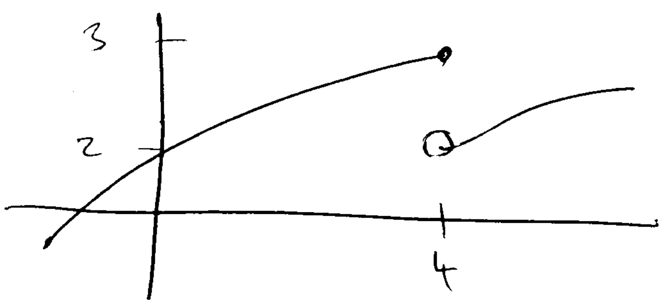
$$\lim_{x \to 5} f(x) = 3 = f(5)$$



$f(x)$

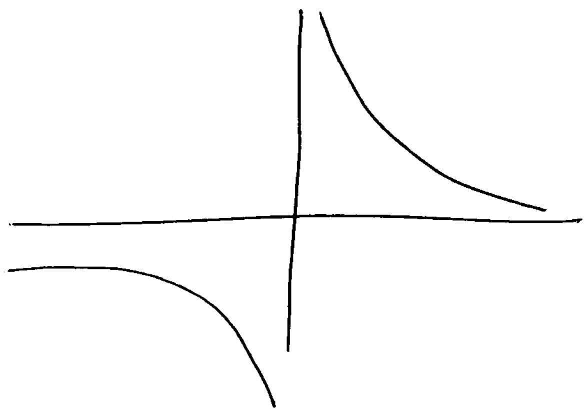
$$\lim_{x \to 5} f(x) = 3$$

$$f(5) = 2$$



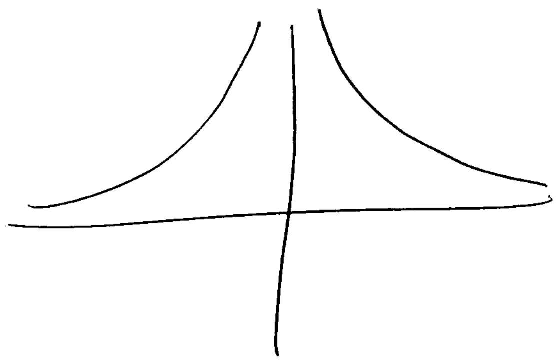
$\lim_{x \to 4} f(x)$ ~~=~~ does not exist

= d.n.e.



$f(x) = \frac{1}{x}$

$\lim_{x \to 0} f(x)$ d.n.e.



$f(x) = \frac{1}{x^2}$

$\lim_{x \to 0} f(x)$ d.n.e.

Limit Laws

- If $f(x) = C$ constant
  then $\lim_{x \to a} f(x) = C$

- If $\lim_{x \to a} f(x) = L$ & $\lim_{x \to a} g(x) = M$
  then $\Rightarrow \lim_{x \to a} f(x) + g(x) = L + M$

  $\Rightarrow \lim_{x \to a} f(x)g(x) = LM$

  $\Rightarrow$ If $M \neq 0$
  $$\lim_{x \to a} \frac{f(x)}{g(x)} = \frac{L}{M}$$

- If n is a positive integer & $a > 0$ if n even
  $$\lim_{x \to a} \sqrt[n]{x} = \sqrt[n]{a}$$

  In <u>particular</u>: $\boxed{\lim_{x \to a} x = a}$

$\boxed{ex}$

$$\lim_{x \to 3} \frac{x-1}{x^2+x+1} = \frac{2}{13}$$

Look at $\lim\limits_{x \to 3} x-1 = 3+(-1) = 2$

$\lim\limits_{x \to 3} x^2+x+1 = 9+3+1 = 13 \neq 0$

$\lim\limits_{x \to 3} x = 3$

$\lim\limits_{x \to 3} (-1) = -1$

$\lim\limits_{x \to 3} x = 3$

$\lim\limits_{x \to 3} 1 = 1$

$\lim\limits_{x \to 3} x^2 = 9$

$\lim\limits_{x \to 3} x = 3$

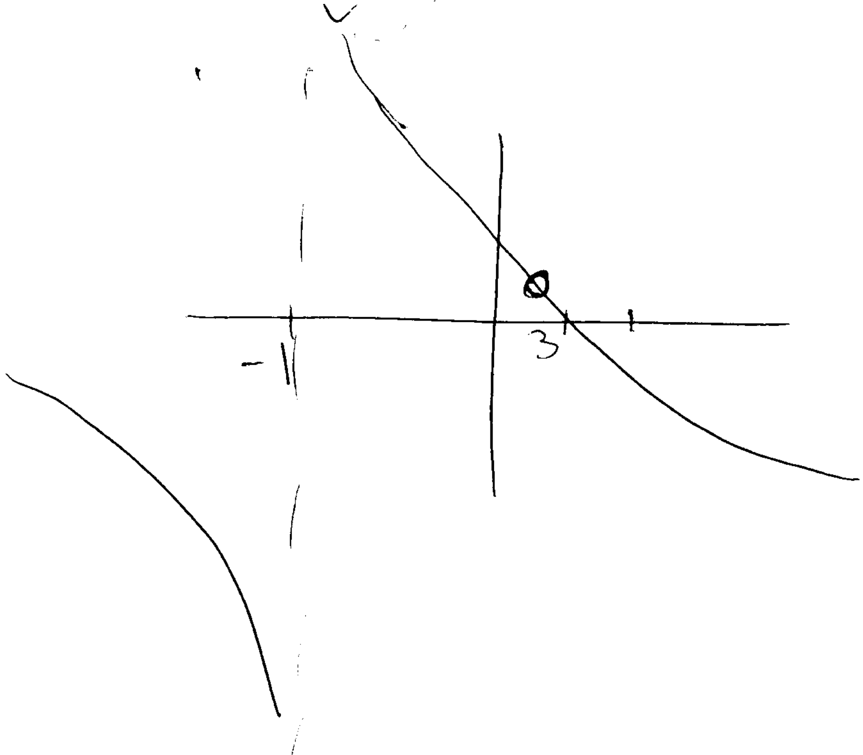
$\lim\limits_{x \to 3} x = 3$

ex

$$\lim_{x\to 3}\frac{x^2-4x+3}{x^2-9} = \lim_{x\to 3}\frac{(x-3)(x-1)}{(x-3)(x+3)} = \frac{3-1}{3+3} = \frac{2}{6}$$

$$\lim_{x\to 3} x^2-4x+3 = 3^2-4(3)+3 = 9-12+3 = 0$$

$$\lim_{x\to 3} x^2-9 = 3^2-9 = 0$$



When can you plug in??

When the function is <u>continuous</u>.

<u>Definition</u> we say a function $f(x)$ is continuous at a point $x=a$ if

$$\lim_{x\to a} f(x) = f(a).$$

<u>examples (tools)</u>

Using limit laws we see that

- $f(x) = \sqrt[n]{x}$ is cont (for $x=a>0$ if n even) for all values of x

- If $f(x)$ & $g(x)$ are cont. at $x=a$ then so are $f+g(x)$ & $f\cdot g(x)$
  (ex if know f, g cont
  $\Rightarrow \lim_{x\to a} f(x) = f(a)$ & $\lim_{x\to a} g(x) = g(a)$
  $\Rightarrow \lim_{x\to a} f+g(x) = f(a)+g(a) = f+g(a)$)

- If $f(x)$ & $g(x)$ cont at $x=a$ & $g(a)\neq 0$ then f/g is cont at $x=a$

Conclusions

Polynomials are continuous!
(So can just plug in to find limits)

Rational functions:

($\frac{p(x)}{q(x)}$ where $p(x)$ & $q(x)$ are polynomials)

these are continuous at any value $x = a$
such that $q(a) \neq 0$

i.e. rat'l functions are continuous anywhere on their domain of definition

Theorem 7 p 93 $\sin x$ & $\cos x$ are continuous everywhere

Heuristic definition a function is continuous if you can draw it without lifting your pen

Substitution Law $\quad f(g(x))$

If $\lim_{x \to a} g(x) = L$ & If $f(x)$ is continuous

at $x = L$ then

$$\lim_{x \to a} f(g(x)) = f(L)$$

$$\left[\lim_{x \to a} f(g(x)) = f\Big(\underbrace{\lim_{x \to a} g(x)}_{L} \Big) \right]$$

<u>ex</u> $\lim_{x \to 5} \cos(3x^2 - 2) = \cos(3 \cdot 5^2 - 2)$

$= \cos(73)$

Interval notation

$[-3, 5]$ = the set of numbers x such that
$-3 \leq x \leq 5$

$(-3, 5)$ = ... $-3 < x < 5$

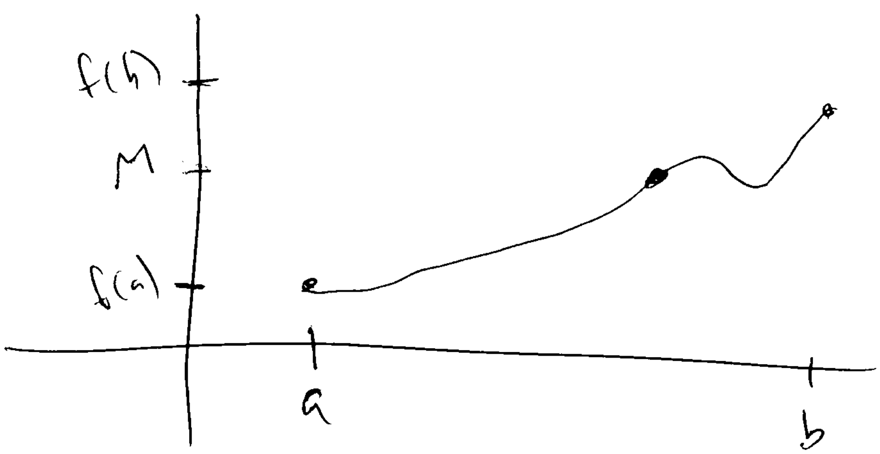
$[-2, 7)$ ...... $-2 \leq x < 7$

$(-\infty, 1]$ $x \leq 1$

$(-\infty, \infty)$ all values of x

Fun with continuity: Intermediate value theorem (IVT)

IVT: Suppose a function $f(x)$ is continuous on an a closed interval $[a,b]$. Then if M is between $f(a)$ & $f(b)$ then there exists c in $[a,b]$ such that $f(c) = M$.



ex $f(x) = 3x^3 - 2x + 6$

solve $f(x) = 0$

$a = -3$

$b = 1$

$f(a) = 3\cdot(-3)^3 - 2(-3) + 6$

$-81 + 6 + 6 = -69$?

$f(b) = 3(1)^3 - 2(1) + 6$

$= 3 - 2 + 6 = 7$

IVT says: somewhere between $x = -3$ & $x = 1$

there is an x value such that $f(c) = 0$

$x = c$