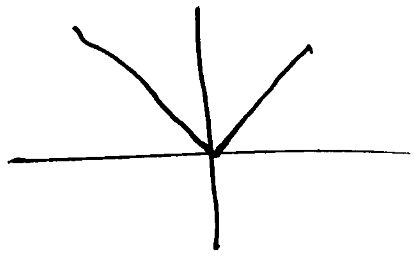
(HW advice)

Remember:

Crit. pts: $\cdot f'(x)$ is not defined on

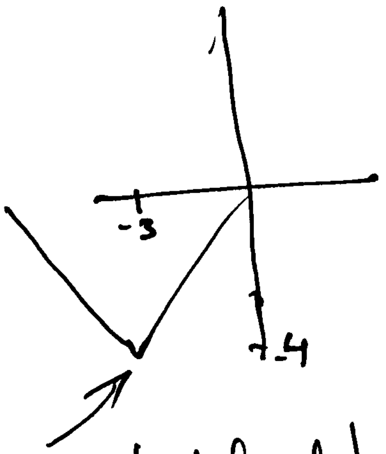
$\cdot f'(x) = 0$.

if $f(x)=|x|$ then $f'(x)$ not defined at $x=0$



$f(x) = |x+3| - 4$

left 3   down 4



$f'(x)$ not defined here $\Rightarrow$ crit pt.

Case 1: $|x|$, der not defined at $x=0$

Case 2: $f'(x) = \frac{1}{x^{1/3}}$

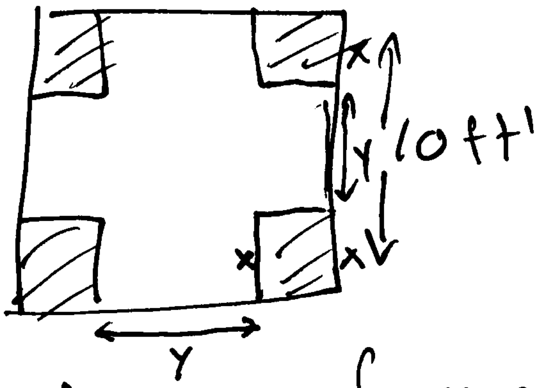
i.e. $f(x) = \frac{3}{2}x^{2/3}$. $f'(x) = \frac{3}{2}\cdot\frac{2}{3}x^{-1/3}$

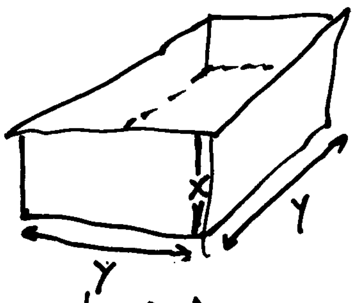
http://www.math.uga.edu/~dkrashen
(course notes)

Applied Optimization

finish 3.5,
read 3.6

Problem Given a square of cardboard



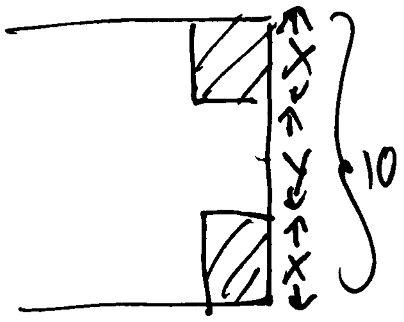


cut of squares from corners & fold up to make a box. What is the largest possible volume of the resulting box?

try to maximize: Volume.

$$V = b \cdot h \cdot w = y \cdot x \cdot y = xy^2$$

use information given to write function (V) as a function of $\underline{1}$ variable.



so $x + y + x = y + 2x = 10$

$$y = 10 - 2x$$

so $V = x(10-2x)^2$

so Volume now a function of x.

$$V(x) = x(10-2x)^2$$

need a closed interval for x:

$$0 \le x \le 5$$

chop off nothing

chop everything

maximize: $V(x) = x(10-2x)^2$ on interval $[0,5]$

$$V'(x) = x \cdot 2(10-2x)^1 \cdot (-2) + (1)(10-2x)^2$$

$$= (10-2x)(-4x + 10 - 2x)$$

$$= (10-2x)(10-6x)$$

Crit pts: $V'(x)$ not defined or (X never happens)

$V'(x) = 0$

$V'(x) = (10-2x)(10-6x) = 0$

$10-2x=0$ or $10-6x=0$

$10=2x$ $\quad$ $10=6x$

$5=x$ $\quad$ $\frac{5}{3}=\frac{10}{6}=x$

list crit pts: $\frac{5}{3}$

end pts: 0, 5

$V(0) = 0$ $\quad$ $V(5/3)$ $\quad$ $V(5)=0$

$V(x) = x(10-2x)^2$

$V(5/3) = \frac{5}{3}\underbrace{(10-2(\tfrac{5}{3}))^2}_{\text{pos}} > 0$ so this is biggest!

$= \frac{5}{3}\left(10-\frac{10}{3}\right)^2 = \frac{5}{3}\left(\frac{20}{3}\right)^2 = \frac{5}{3}\cdot\frac{400}{9}$

$= \frac{2000}{27} \approx 74.$

Procedure:

1: Read problem

2: Determine what you are trying to maximize or minimize

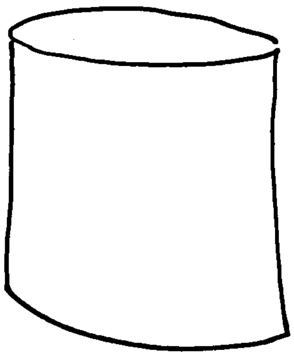
3: Give variable names to quantities to write an equation for what you are trying to max/minimize

4: Use problem info to eliminate variables (if necessary) to get a fcn of <u>1</u> variable.

5: Determine a closed interval for domain (how big/small does variable get?)

6: Do it. (section 3.5)

Building a can



Want to build a can at a total cost of 300π cents

top & bottom made from copper at a cost of 2¢$/in^2$

sides made of aluminum at a cost of 1¢$/in^2$

maximize volume of can

(volume of a cylinder is $V = \pi r^2 h$

cost of top + cost of bott + cost of sides $= 300\pi$

$2 \cdot$ (area of top) $+ 2$(area of bottom) $+ 1$(area of sides) $= 300\pi$

$$2\pi r^2 + 2\pi r^2 + 2\pi r h = 300\pi$$

$$4\pi r^2 + 2\pi r h = 300\pi$$

$V = \pi r^2 h$

r & h related via

$$4\pi r^2 + 2\pi r h = 300\pi$$

solve for h

$$2\pi r h = 300\pi - 4\pi r^2$$

$$h = \frac{300\pi}{2\pi r} - \frac{4\pi r^2}{2\pi r}$$

$$\boxed{h = \frac{150}{r} - 2r}$$

$$V = \pi r^2\left(\frac{150}{r} - 2r\right)$$

$$= 150\pi r - 2r^3\pi = 2r\pi(75 - r^2)$$

$$V(r) = 2\pi r(75 - r^2)$$

$$= 150\pi r - 2\pi r^3$$

interval for r:

$$0 \le r \le \sqrt{75}$$

$h=0$ $\rightarrow$ $0 = \frac{150}{r} - 2r$

$$2r = \frac{150}{r}$$

$$r^2 = 75$$

$$r = \pm\sqrt{75}$$

max. $V(\cancel{x} r)$ on the interval $[0, \sqrt{75}]$

$V(r) = 2\pi r(75 - r^2)$ on

$= 150\pi r - 2\pi r^3$

$V'(r) = 150\pi - 6\pi r^2 = 0$

$6\pi r^2 = 150\pi$

$r^2 = \frac{150\pi}{6\pi} = \frac{150}{6} = \frac{75}{3} = 25$

$r = 5$

so need to check

crit pts : $(r = 5)$

end pts : $r = 0$, $r = \sqrt{75}$

$V(0) = 0$

$V(\sqrt{75}) = 0$

$V(5) = 2\pi(5)(75 - 5^2) = 10\pi(75 - 25)$

max volume (at $r = 5$) $\rightarrow$ $\boxed{500\pi}$