Derivatives of logarithms with other bases

Recall: $\frac{d}{dx}(\ln x) = \frac{1}{x}$

What about $\frac{d}{dx}(\log_a x)$ for $a \neq e$?

for example $\frac{d}{dx}(\log_3 x)$?

Answer: Change of base formula:

$$\log_a x = \frac{\log_b x}{\log_b a} \quad \text{(to change from base a to base b)}$$

Why? suppose $y = \log_a x$ i.e. $a^y = x$.

taking $\log_b$ of both sides,

$$\log_b a^y = \log_b x$$

$$y \log_b a = \log_b x \qquad y = \log_a x$$

$$(\log_a x)(\log_b a) = \log_b x$$

Now, dividing by $\log_b a$ gives

$$\boxed{\log_a x = \frac{\log_b x}{\log_b a}}$$

In particular, to change from base a to base e:

$$\log_a x = \frac{\ln x}{\ln a}$$

So $\frac{d}{dx} \log_3 x = \frac{d}{dx}\left(\frac{\ln x}{\ln 3}\right) = \frac{1}{\ln 3}\frac{d}{dx}(\ln x)$

$$= \frac{1}{\ln 3} \frac{1}{x} = \frac{1}{x \ln 3}.$$

In general, $\frac{d}{dx} \log_a x = \frac{d}{dx}\left(\frac{\ln x}{\ln a}\right) =$

$$\frac{1}{\ln a} \cdot \frac{1}{x} = \frac{1}{x \ln a}.$$