Here's a few drill problems. These equations are all linear first order ordinary differential equations, or convertable to such. The answers were provided by Maple and may not be in exactly the form you will obtain. The Maple work is available on my website as a Maple worksheet: `256w2002-linear-1-ode.mws`

Problem 0120 – 1.

$$\frac{dy}{dx} - 4xy = 2x$$

Ans: $y = -\frac{1}{2} + C\mathrm{e}^{2x^2}$

Problem 0120 – 2.

$$\frac{du}{dt} = \frac{u}{t} + t^5$$

Ans: $u = \left(\frac{1}{5}t^5 + C\right) t$

Problem 0120 – 3.

$$\frac{dx}{dt} + \sec(t)x = \tan(t)$$

Ans: $x = \dfrac{1 + \sin(t) - t\cos(t) + C\cos(t)}{1 + \sin(t)}$

Problem 0120 – 4.

$$\frac{dx}{dt} + \tan(t)x = \sec(t)$$

Ans: $x = \sin(t) + C\cos(t)$

Problem 0120 – 5.

$$\frac{dy}{dx} = \frac{y}{x+y}$$

This equation is not linear but the chain rule allows us to convert it to a linear ODE for x in terms of y.

Ans: $x = (\log(y) + C)\, y$

Problem 0120 – 6. A tank is full of brine of concentration 2 g/L salt. Fresh water flows into the tank at 3 L/min and the well-mixed solution is drawn off at the same rate. After 15 minutes the concentration of brine in the outflow from the tank is 1.2 g/L. Find the volume of the tank.

Ans: 88.09 L

Problem 0120 – 7.

$$t\frac{dx}{dt} + 4x = \cos(t), \quad x\left(\frac{\pi}{2}\right) = \frac{2}{\pi}$$

Ans: $x = \dfrac{t^3\sin(t) + 3t^2\cos(t) - 6\cos(t) - 6t\sin(t) + 3\pi}{t^4}$

Problem 0120 – 8.

$$\frac{dy}{dt}\cos(t) + \sin(t)y = \sin(t), \quad y(0) = \pi$$

Ans: $y = 1 + \cos(t)\,(-1 + \pi)$

Problem 0120 – 9.

$$\left(1 + x^2\right)\frac{dy}{dx} + 2xy = 3 + 3x + 2x^2 + x^3$$

Ans: $y = \dfrac{36x + 18x^2 + 8x^3 + 3x^4 + C}{12\left(1 + x^2\right)}$

Problem 0120 – 10.

$$(1 + x)\frac{dy}{dx} + y = \sin(x) + \cos(x)$$

Ans: $y = \dfrac{\sin(x) - \cos(x) + C}{1 + x}$

Problem 0120 – 11.

$$\left(1 - xy^2\right)\frac{dy}{dx} = y^3$$

This equation is not linear but the chain rule allows us to convert it to a linear ODE for x in terms of y.

Ans: $xy^2 = Cy^2 - 1$

Problem 0120 – 12.

$$\left(t^2 + 1\right)\frac{dx}{dt} + 3t^3x = 5t^3\mathrm{e}^{-3t^2/2}, \quad x(0) = 2$$

Ans: $x = \dfrac{16}{3}\left(t^2 + 1\right)^{3/2}\mathrm{e}^{-3t^2/2} - \dfrac{5}{3}\left(3t^2 + 2\right)\mathrm{e}^{-3t^2/2}$