Math 2310H, Fall 2002, First Exam

Directions: This exam is worth 100 points. You will have one hour to complete this exam. You may **not** use calculators, but they will not be necessary. Please show all of your work. If you need scratch paper, please use the extra page at the end of this exam. This exam should have 8 pages. *Good luck!*

Name:

Problem	Possible	Score
1	15	
2	15	
3	15	
4	15	
5	20	
6	20	
Total	100	

1. (15 points) This problem has two parts. The first part is worth five points. The second part is worth ten points.

(a) Compute the integral

$$\int_0^{\pi/2} \sin^3(x)\cos(x)\,dx$$

(b) Compute the integral

$$\int_0^{\pi} \cos^4(x)\,dx$$

2. (15 points) This problem has two parts. The first part is worth five points. The second part is worth ten points.

(a) Compute the derivative

$$\frac{d}{dx}\int_3^x \sin(\sqrt{t+2})\,dt$$

(b) Compute the derivative

$$\frac{d}{dx}\int_3^{\cos x} \sin(\sqrt{t+2})\,dt$$

3. (15 points) Find the volume obtained when the region under the curve $y = x^2$, $0 \leq x \leq 2$ is rotated around the $y-$axis.

4. (15 points) Find the arclength of the curve $y = x^{3/2}$, $0 \leq x \leq 3$.

5. (20 points) This problem has two parts. Each is worth ten points.

(a) Find the area between the curves $y = x^2$ and $y = x^3$, $0 \leq x \leq 2$.

(b) Write down an integral for the volume obtained when the area under the curve $y = \sin(x)$, $0 \leq x \leq \pi$ is rotated around the line $x = -2$.

6. (20 points) This problem has three parts. The first part is worth ten points. The second and third parts are worth five points each.

(a) Write down an integral for the surface area generated when the curve $y = x^3$, $0 \leq x \leq 1$ is rotated around the line $x = 7$.

(b) Suppose $0 < a < b$ are constants (what they are is not relevant to the problem - this is just a technical detail so the rest of it works). Find a relationship between the constants A and B so that you can evaluate the integral of the arclength of $y = Ax^4 + \dfrac{B}{x^2}$, $a \leq x \leq b$ without having to do an integral involving square roots (i.e. find some way to make the square root in the formula go away!).

(c) Find the value of

$$\lim_{n\to\infty} \sum_{k=0}^{n-1} \sqrt{1 + k\frac{2}{n}}\ \frac{2}{n}.$$

(scratch page)