Math 3000 (Lieman), Fall 2002, First Exam

Directions: This exam is worth 100 points. You will have one hour to complete this exam. You may **not** use calculators, but they will not be necessary. Please show all of your work. If you need scratch paper, please use the extra page at the end of this exam. This exam should have 7 pages. *Good luck!*

Name:

Problem	Possible	Score
1	20	
2	20	
3	20	
4	50	
5	50	
Total	100	

1. (20 points) This problem has two parts, each worth ten points.

(a) Find a nonzero vector $\overrightarrow{\mathbf{x}}$ (in $\mathbb{R}^3$) perpendicular to the vector $\overrightarrow{(1,2,3)}$.

(b) Find a vector $\overrightarrow{\mathbf{v}}$ that points in the same direction as $\overrightarrow{\mathbf{x}}$ (the vector you found in part (a)) but with $\|\overrightarrow{\mathbf{v}}\| = 1$.

2. (20 points) This problem has three parts. The first is worth five points. The second is worth ten points. The third is worth five points.

(a) Find the normal vector $\overrightarrow{\mathbf{n}}$ to the plane given by $x_1 - x_2 + x_3 = 7$.

(b) Find the distance from the point $(3, 2, 3)$ to the plane in part (a).

(c) Find the closest point on this plane (in part (a)) to $(3, 2, 3)$.

3. (20 points) This problem has three parts. The first is worth five points. The second is worth ten points. The third is worth five points.

(a) Write down the augmented matrix associated to the system

$$\begin{aligned} x_1 + x_2 + x_3 &= 3 \\ 2_1 + 2x_2 + 3x_3 &= 9 \\ 3x_1 + 3x_2 + 4x_3 &= 12 \end{aligned}$$

(b) Find the reduced row echelon form of the matrix in part (a).

(c) Write down the general linear solution to the system in part (a).

4. (20 points) This problem has four parts. Each part is worth three points. You must justify your answers! The first three parts concern the matrix

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & -1 \end{pmatrix}$$

a) How many pivots does A have?

b) Suppose we have a linear system $A\overrightarrow{\mathbf{x}} = \overrightarrow{\mathbf{b}}$. What must the size of the (column) vectors $\overrightarrow{\mathbf{x}}$ and $\overrightarrow{\mathbf{b}}$ be?

c) Find a 3×2 matrix B (by guessing, trial and error) such that $AB = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$.

d) True or false: Every 2×3 matrix A has a 3×2 inverse B so that $AB = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$. If you think it is false, you must provide a matrix A for which no inverse exists, and explain why no inverse exists. If you think it is true, you must explain why.

5. (20 points) This problem has two parts. Each part is worth ten points.

a) For what value of α is the vector $\overrightarrow{(\frac{3}{5}, \alpha)}$ a **unit** vector?

b) For what value(s) of α does the matrix $\begin{pmatrix} 1 & \alpha \\ \alpha & 3\alpha \end{pmatrix}$ have rank 1 (for most values of α, it has rank 2 - put it in row echelon form and see what you can figure out)?

(scratch page)