Math 3000 (Lieman), Fall 2002, Second Exam

Directions: This exam is worth 100 points. You will have one hour to complete this exam. You may **not** use calculators, but they will not be necessary. Please show all of your work. **You must justify your answers to get full credit on yes/no questions.** If you need scratch paper, please use the extra page at the end of this exam. This exam should have 11 pages. *Good luck!*

Name:

Problem	Possible	Score
1	20	
2	20	
3	20	
4	20	
5	20	
Total	100	

1. (20 points) This problem has two parts, each worth ten points. For each part, determine whether the indicated subset is a vector subspace. You must justify your answer (as on every question on this test!) in order to get full credit.

(a) The set of vectors $\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} \in \mathbb{R}^4$ that satisfy $x_1+x_2+x_3 = 0$ and $x_1+x_3+x_4 = 0$.

1 (continued). (b) The set of vectors $\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} \in \mathbb{R}^4$ that satisfy $x_1 + x_2 + x_3 = 0$ and $x_1 + x_3 x_4 = 0$.

2. (20 points) This problem has four parts, each worth five points.

(a) Are the two vectors $\overrightarrow{\mathbf{v}}_1 = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$ and $\overrightarrow{\mathbf{v}}_2 = \begin{pmatrix} 2 \\ 4 \end{pmatrix}$ linearly independent?

(b) Do the two vectors $\overrightarrow{\mathbf{v}}_1$ and $\overrightarrow{\mathbf{v}}_2$ of part (a) span $\mathbb{R}^2$? Why or why not?

2 (continued)

(c) Let A be the 3×3 matrix $\begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 6 \\ 5 & 2 & 9 \end{pmatrix}$. What is the rank of A? What is the dimension of the kernel of A?

(d) Find a basis for the subspace (of $\mathbb{R}^4$) spanned by $\begin{pmatrix} 1 \\ 3 \\ 2 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 4 \\ 1 \end{pmatrix}, \begin{pmatrix} 2 \\ 4 \\ 6 \\ 2 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \\ 4 \\ 1 \end{pmatrix}$.

3. (20 points)

Are the vectors $\begin{pmatrix}1\\2\\3\\4\end{pmatrix}, \begin{pmatrix}1\\3\\4\\3\end{pmatrix}, \begin{pmatrix}1\\2\\3\\0\end{pmatrix}, \begin{pmatrix}1\\1\\2\\1\end{pmatrix}$ a basis for $\mathbb{R}^4$?

4. (20 points) This problem has three parts. The first two parts are worth five points each. The third part is worth ten points.

(a) Suppose A is an invertible matrix such that $A^T = A^{-1}$. Show that $A\overrightarrow{\mathbf{x}} \cdot A\overrightarrow{\mathbf{y}} = \overrightarrow{\mathbf{x}} \cdot \overrightarrow{\mathbf{y}}$ for all pairs of vectors $\overrightarrow{\mathbf{x}}, \overrightarrow{\mathbf{y}}$.

(b) Find the transpose of the matrix $\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix}$.

4 (continued)

(c) Find the inverse of the matrix $\begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 3 \\ -1 & 0 & 7 \end{pmatrix}$.

5. (20 points) This problem has three parts. The first two parts are worth five points each. The third part is worth ten points.

(a) Suppose A is an $n \times n$ matrix and $\overrightarrow{\mathbf{x}}$ and $\overrightarrow{\mathbf{y}}$ are nonzero vectors in $\mathbb{R}^n$ with

$$A\overrightarrow{\mathbf{x}} = 2\overrightarrow{\mathbf{x}}$$
$$A\overrightarrow{\mathbf{y}} = 3\overrightarrow{\mathbf{y}}.$$

Show that $\overrightarrow{\mathbf{x}}$ and $\overrightarrow{\mathbf{y}}$ are linearly independent.

5 (continued)

(b) Give an example of a linearly independent set of vectors in $\mathbb{R}^3$ that does not span $\mathbb{R}^3$.

(c) Oh, what the heck. The football team *is* undefeated. Let's celebrate at least a little bit. You don't have to do anything for this problem. You get ten points just for showing up.

(scratch page)