Math 3000 (Lieman), Fall 2002, Third Exam

Directions: This exam is worth 100 points. You will have one hour to complete this exam. You may **not** use calculators, but they will not be necessary. Please show all of your work. **You must justify your answers to get full credit on yes/no questions.** If you need scratch paper, please use the extra page at the end of this exam. This exam should have 8 pages. *Good luck!*

Name:

Problem	Possible	Score
1	20	
2	20	
3	20	
4	20	
5	20	
Total	100	

1. This problem is worth 20 points. The first two parts are worth 5 points each. The third part is worth 10 points.

a) Find a vector $\vec{\mathbf{v}}$ so that the following set is orthogonal (in $\mathbb{R}^3$)

$$\left\{ \begin{pmatrix} \frac{1}{3\sqrt{2}} \\ \frac{1}{3\sqrt{2}} \\ -\frac{4}{3\sqrt{2}} \end{pmatrix}, \begin{pmatrix} \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \\ 0 \end{pmatrix}, \vec{\mathbf{v}} \right\}.$$

b) Find a vector $\vec{\mathbf{v}}$ so that the list of vectors given in part a) is orthonormal.

1. (continued)

c) Let L be the linear transformation $L : \mathbb{R}^2 \to \mathbb{R}^2$ which acts by projecting a vector onto the x-axis. I.e. $L(\overrightarrow{\mathbf{x}})$ is the projection of $\overrightarrow{\mathbf{x}}$ onto the x-axis. Find a matrix A such that $A(\overrightarrow{\mathbf{x}}) = L(\overrightarrow{\mathbf{x}})$.

2. This problem is worth 20 points.

Use the Gram-Schmidt process to find an orthonormal set of vectors with the same span as the set

$$\left\{ \begin{pmatrix} 1 \\ 7 \\ 1 \\ 7 \end{pmatrix}, \begin{pmatrix} 0 \\ 7 \\ 2 \\ 7 \end{pmatrix}, \begin{pmatrix} 1 \\ 8 \\ 1 \\ 6 \end{pmatrix} \right\}.$$

3. This problem is worth 20 points.

Find the least squares solution $\overrightarrow{\mathbf{x}}$ to the inconsistent system $A\overrightarrow{\mathbf{x}} = \overrightarrow{\mathbf{b}}$, where $A = \begin{pmatrix} 1 & 1 \\ 1 & 0 \\ 0 & 1 \end{pmatrix}$ and $\overrightarrow{\mathbf{b}} = \begin{pmatrix} 3 \\ 3 \\ 3 \end{pmatrix}$.

4. This problem is worth 20 points.

Let $L : \mathbb{R}^3 \to \mathbb{R}^3$ be the linear transformation given by reflecting across the plane $-x_1 + x_2 + x_3 = 0$. Find the representation of L with respect to the standard basis. (Hint: you may first want to choose a basis of $\mathbb{R}^3$ for which it is easy to compute the action of L and then use a change of basis formula to get the final answer.)

5. This problem is worth 20 points.

Find the projection of the vector $\begin{pmatrix} 1 \\ 4 \\ 3 \end{pmatrix}$ onto the plane spanned by the two vectors $\begin{pmatrix} 1 \\ -3 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}$.

(scratch page)