Name GM

ID#

Math 2250
Exam 2 practice, fall 2011
Prof G.Matić

1. Find $\frac{dy}{dx}$ for the functions below. ~~Do not~~ S simplify.

(a) $y = \cos^3(3x^2)$

$y = (\cos(3x^2))^3$

$y' = 3 \cdot (\cos(3x^2))^2 \cdot (\cos(3x^2))' =$

$= 3\cos^2(3x^2) \cdot (-\sin(3x^2)) \cdot 6x$

$= -18x \sin(3x^2)\cos^2(3x^2)$

(b) $y = e^{5x}\, sec^3 x$

$y' = (e^{5x})' \cdot (\sec^3 x) + e^{5x} \cdot (\sec^3 x)'$

$= 5e^{5x} \cdot \sec^3 x + e^{5x} \cdot 3 \cdot \sec^2 x \cdot (\sec x)'$

$= 5e^{5x} \cdot \sec^3 x + 3e^{5x} \cdot \sec^2 x \cdot \sec x \cdot \tan x$

$= \sec^3 x \,(5e^{5x} + 3e^{5x}\tan x) = e^{5x}\sec^3 x\,(5 + 3\tan x)$

(c) $y = \ln(\cos(1 - x^2))$

$y' = \frac{1}{\cos(1-x^2)} \cdot (\cos(1-x^2))'$

$= \frac{1}{\cos(1-x^2)} \cdot (-\sin(1-x^2)) \cdot (-2x)$

$= \frac{2x\sin(1-x^2)}{\cos(1-x^2)} = 2x\tan(1-x^2)$

(d) $y = \sin^{-1}(e^x)$

$$y' = \frac{1}{\sqrt{1-(e^x)^2}} \cdot (e^x)' = \frac{e^x}{\sqrt{1-e^{2x}}}$$

(e) $y = x^{\sin x}$

Since x is in numerator and denominator, have to use logar. diff

$\ln y = \ln x^{\sin x}$ exp property of logs

$\ln y = \sin x \cdot \ln x$ do $\frac{d}{dx}$ of both sides, use product rule

$\frac{1}{y}\frac{dy}{dx} = \cos x \cdot \ln x + \sin x \cdot \frac{1}{x}$ multiply by y

$\frac{dy}{dx} = y \cdot \left(\cos x \cdot \ln x + \frac{1}{x} \sin x \right)$

$\frac{dy}{dx} = x^{\sin x}\left(\cos x \cdot \ln x + \frac{1}{x}\sin x\right)$

(f) Use logarithmic differentiation to find y' for $y = \dfrac{(x^2-4)^2(x^2+3)^3}{(3x-1)^2(x-2)^5}$

$$\ln y = 2\ln(x^2-4) + 3\ln(x^2+3) - 2\ln(3x-1) - 5\ln(x-2)$$

$$\frac{1}{y}\frac{dy}{dx} = 2 \cdot \frac{1}{x^2-4} \cdot 2x + 3\,\frac{1}{x^2+3}\,2x - 2 \cdot \frac{1}{3x-1} \cdot 3 - 5 \cdot \frac{1}{x-2}$$

$$\frac{dy}{dx} = y \cdot \left[\frac{4x}{x^2-4} + \frac{6x}{x^2+3} - \frac{6}{3x-1} - \frac{5}{x-2} \right]$$

$$\frac{dy}{dx} = \frac{(x^2-4)^2(x^2+3)^3}{(3x-1)^2(x-2)^5}\left(\frac{4x}{x^2-4} + \frac{6x}{x^2+3} - \frac{6}{3x-1} - \frac{5}{x-2} \right)$$

3. Find the equation of the line tangent to the curve given by

$$x^2y^3 = cos(y^2 - x^2)$$

at the point (1,1) on the curve.

Implicit differentiation

$$2x \cdot y^3 + x^2 \cdot 3y^2 \cdot \frac{dy}{dx} = -\sin(y^2-x^2) \cdot \left[2y\frac{dy}{dx} - 2x\right]$$

$$2xy^3 + 3x^2y^2\frac{dy}{dx} = -2y\sin(y^2-x^2)\frac{dy}{dx} + 2x\sin(y^2-x^2)$$

$$3x^2y^2\frac{dy}{dx} + 2y\sin(y^2-x^2)\frac{dy}{dx} = 2x\sin(y^2-x^2) - 2xy^3$$

$$\frac{dy}{dx} \cdot y \cdot \left(3x^2 + 2\sin(y^2-x^2)\right) = 2x\left(\sin(y^2-x^2) - y^3\right)$$

$$\frac{dy}{dx} = \frac{2x\left(\sin(y^2-x^2) - y^3\right)}{y\left[3x^2 - 2\sin(y^2-x^2)\right]}$$

$$m_{tan} = \left.\frac{dy}{dx}\right|_{(1,1)} = \frac{2 \cdot 1\left[\sin 0 - 1\right]}{1 \cdot \left[3 - 2\sin 0\right]}$$

$$m_{tan} = \frac{2}{3}$$

$$y - 1 = \frac{2}{3}(x-1)$$

$$y = \frac{2}{3}x - \frac{2}{3} + 1$$

$$\boxed{y = \frac{2}{3}x + \frac{1}{3}}$$

4. A ball is dropped from a building 400 ft high. How long does it take it to reach the ground. With what velocity will it hit the ground?

$$s(t) = -16t^2 + 0 \cdot t + 400$$

When does it reach the ground:

when $s(t) = 0$

$$-16t^2 + 400 = 0$$

$$400 = 16t^2$$

$$100 = 4t^2$$

$$t^2 = 25 \qquad \boxed{t = 5}$$

What is $v(t)$ when it hits the ground!

$$v(t) = -32t$$

$$v(5) = -32 \cdot 5 = -160$$

$$\boxed{v = -160}$$

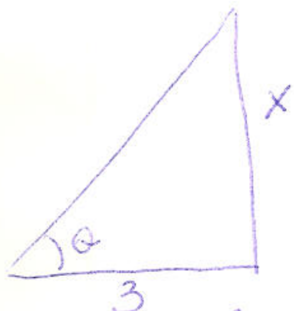
YOU WOULD NEED TO DO 2 OF THESE KINDS OF PROBLEMS

5. An observer on the ground is watching a rocket that was launched vertically. He noticed that the elevation angle θ of the line of sight to the rocket (relative to the ground) was increasing $\frac{1}{24}$ radians per second when $\theta = \frac{\pi}{3}$. Knowing that the observer was 3 miles away from the launch pad, find the speed of the rocket at that instant.

6. Coffee is draining from a conical filter into a cylindrical coffeepot at the rate of 10 cm^3/min. The filter is 10 cm tall and is also 10 cm in diameter at the top. The coffeepot fits under it and is 10 cm in diameter as well. How fast is the level of the liquid falling in the cone, when it is 5 cm deep (in the cone). How fast is the level of the coffee rising in the pot at that moment?

7. Air is escaping from a spherical balloon at a constant rate of 100π in^3/s. What is the radius of the balloon when its radius is decreasing at the rate of 3 in/s?

5)



moving ↑

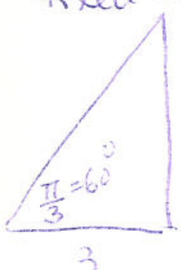
$\frac{X}{3} = \tan\theta$

$X = 3\tan\theta$

$\frac{dx}{dt} = 3 \cdot \sec^2\theta \cdot \frac{d\theta}{dt}$

$\frac{dx}{dt} = 3 \left(\frac{1}{\cos\theta}\right)^2 \cdot \frac{d\theta}{dt}$

Fixed time ↓



$x = 3\tan\frac{\pi}{3}$
$= 3\sqrt{3}$!

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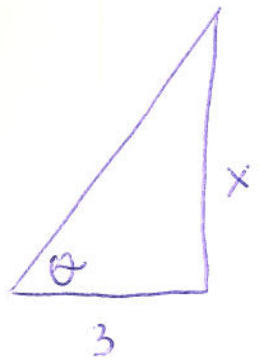
another way to think of it is 60-30-90 triangle

$\sqrt{36 - 9} =$
$\sqrt{27} =$
$3\sqrt{3}$

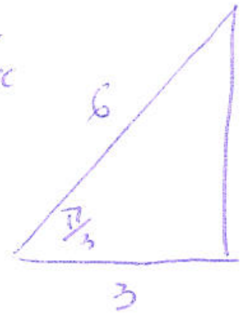
see next page

5)

1) 

$\frac{d\theta}{dt} = \frac{1}{24}$ rad/sec

when $\theta = \frac{\pi}{3}$



$\sqrt{36-9} = \sqrt{27} = 3\sqrt{3}$

2) $\frac{x}{3} = \tan\theta$

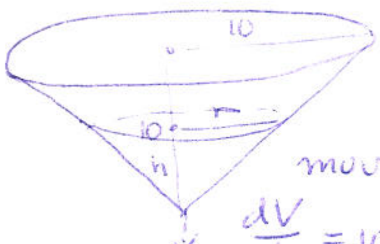
$x = 3\tan\theta$

3) $\frac{dx}{dt} = 3\sec^2\theta \frac{d\theta}{dt}$

4) when $\theta = \frac{\pi}{3}$, $\sec\theta = \frac{1}{\cos\theta} = \frac{1}{\cos\frac{\pi}{3}} = \frac{1}{\frac{1}{2}} = 2$

so we have

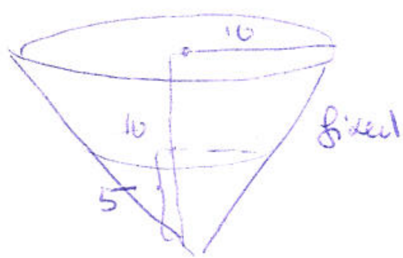
$\frac{dx}{dt} = 3\cdot(2)^2\cdot\frac{1}{24} = \cancel{3}\cdot 4\cdot\frac{1}{\cancel{24}_{\cancel{6}}} = \frac{1}{2}$ mi/sec

6) ① 

moving

$\frac{dV}{dt} = 10$ cm³/min

$\frac{r}{h} = \frac{10}{10}$ so $r = h$ at all times



fixed

② $V = \frac{1}{3}\pi r^2 h$

$r = h$ so

$V = \frac{1}{3}\pi h^3$

③ $\frac{dV}{dt} = \frac{1}{\cancel{3}}\pi\cdot\cancel{3}h^2\cdot\frac{dh}{dt}$

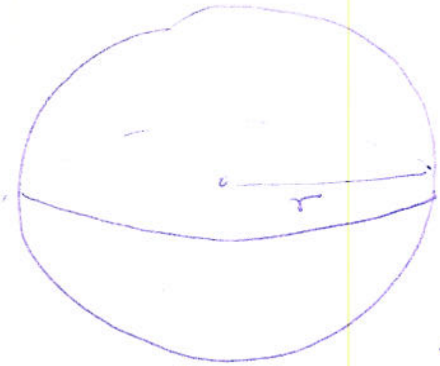
$\frac{dV}{dt} = \pi h^2 \frac{dh}{dt}$

when $h = 5$ ④ $10 = \pi\cdot 25\cdot\frac{dh}{dt}$

$\frac{dh}{dt} = \frac{10}{\pi\cdot 25} = \boxed{\frac{2}{5\pi}}$ cm/min

7.

1



$$\frac{dV}{dt} = 100\pi \ in^3/s$$

$$\frac{dr}{dt} = 3 \ in/s$$

what is r

(No fixed picture here, they did not give us r)

2 $V = \frac{4}{3}\pi r^3$

3 $\frac{dV}{dt} = \frac{4}{\cancel{3}}\pi \cdot \cancel{3} r^2 \frac{dr}{dt}$

$$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$$

$$100\pi = 4\pi r^2 \cdot 3$$

$$\frac{\overset{25}{\cancel{100}}\cancel{\pi}}{\cancel{4}\cdot 3\ \cancel{\pi}} = r^2$$

$$r^2 = \frac{25}{3}$$

$$r = \sqrt{\frac{25}{3}} = \frac{5}{\sqrt{3}}$$

$$\boxed{r = \frac{5\sqrt{3}}{3}}$$