Summary of Research

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For the previous forty years, theory of finance has been emerging to a new scientific discipline. Since the Black-Scholes model was published in 1973, mathematical finance has become a branch of applied mathematics concerned with financial markets. The original model that Black and Scholes used to describe the behavior of asset price was called the geometric Brownian motion (GBM) which can be represented by a stochastic differential equation $dX_t = \mu X_t dt + \sigma X_t dW_t$, where X_t is the price of an asset under consideration, μ is the expected return of the asset, σ is the volatility of the asset price and W_t is a standard Brownian motion. Due to the complexity of the financial world, the GBM is not sufficient to capture the market behaviors. As a result, various models have been developed in the past few decades.

My research concerns about optimal trading on two different models. The first project provided an optimal trading rule for an asset which has price governed by the mean-reversion model. The results led to a journal publication in the Discrete and Continuous Dynamical System Series B. In the second project, I discovered sufficient conditions for the possibility of optimal trading on a trend-following strategy, which resulted in another journal paper accepted by Automatica: A Journal of IFAC. Finally, I improved the practicality of the first project by implementing stochastic approximation algorithm to reduce computation time and weaken model specifications.

1 An Optimal Trading Rule of a Mean-Reverting Asset

A mean-reversion model is often used in financial and energy markets to capture price movements that have the tendency to move towards an "equilibrium" level. The behavior of the model can be described by a stochastic differential equation

$$dX_t = a(L - X_t)dt + \sigma dW_t \quad (1)$$

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where a is the reversion rate and L is the equilibrium level; X_t is log of the price of an asset, σ and W_t are the same as those in the GBM above. I studied the trading rule on an asset such that the expected profit is maximized. In particular, the trading rule involves three aspects: buying, selling and shorting. Shorting or short selling is the practice of selling financial securities the seller does not then own, in the hope of repurchasing them later at a lower price. A typical trading strategy in financial markets is to buy low then sell high, or sell high then buy low if one is selling short. However, identifying these buy and sell prices poses a great challenge. The objective is to determine those threshold price levels when the behavior of the asset price follows a mean-reversion model.

A significant volume of literature was concerned with trading rules in financial markets; see for instance, Zhang [3] and Guo and Zhang [2]. Treating a mean-reverting asset, Zhang and Zhang [4] was devoted to optimum trading strategy. It established two threshold prices (buy and sell) that maximize overall discounted return if one trades at those prices. Nonetheless, the net positions of its formulation are limited to either flat or long. In other words, short selling, a common operation in stock market, was not included in their studies.

In order to obtain a more realistic trading rule, short selling is taken into account in my formulation. In addition, I allow either one share long, or flat, or one share short at any given time. One is allowed to choose between shorting and buying when one has no share in holding. In this paper, I also consider slippage costs associated with each transaction because it becomes noteworthy in frequent transactions. In my formulation, a fixed rate of slippage cost is incurred in each transaction. The objective is to buy, sell or short so as to maximizing a discounted reward function. I follow a dynamic programming approach to resolve the problem and obtain the corresponding Hamilton-Jacobi- Bellman (HJB) equations for the value functions. Using these HJB equations, I solve the optimal stopping problem by determining four threshold levels corresponding to buying and selling points. These levels are then used to convert the HJB equations into quasi-algebraic equations via a smooth-fit technique. I also provide a verification theorem to assure the optimality of the trading rule.

2 A Trend Following Strategy: Conditions for Optimality

In the celebrated Black-Scholes model, the GBM used is a stochastic differential equation with constant expected return and volatility. It gives a reasonable account of the market behavior in a short period of time. Nevertheless, it fails to describe the asset prices in the long run. The reason for that is the incapability of response to the market changes. The regime switching model was developed to fix that shortcoming to some extend. In other words, it is a GBM with varying parameters.

The regime switching model that I work on is a stochastic differential equation that similar to GBM, $dX_t = \mu(\alpha_t)X_t dt + \sigma(\alpha_t)X_t dW_t$, where α_t is a continuous time Markov chain with two states, namely bull market ($\alpha_t = 1$) and bear market ($\alpha_t = 2$). Clearly, $\mu(1)$ is positive and $\mu(2)$ is negative. Assume that α_t's are observable, i.e. one can tell when the market switches between bull and bear. Then the optimal trading strategy is to buy at the switching time of bear-to-bull and sell at the switching time of bull-to-bear, which is known as trend following. However, it is not always profitable. Consider a simple case; if the switching between bull and bear is too frequent and the slippage cost is not trivial, the total cost on transactions becomes significant enough that may offset the revenue from trading. I am able to determine explicitly the sufficient conditions on market parameters that make optimal trading possible. Again, a verification theorem is proved to assure the optimality.

3 Stochastic Approximation on Trading Mean-Reverting Assets

The first paper is an improvement of Zhang and Zhang [4]. However, for the feasibility of actual implementation on the real markets, a great deal of refinement is still needed. One of the biggest weaknesses of the method used in the paper is the parameters calibration. All the parameters on (1) have to be estimated appropriately in order to compute the results. The performance of the method is highly dependent on the accuracy of estimates. A way to overcome this defect is by using stochastic approximation approach. The mean-reversion model is still assumed, however one of the advantages is that the underlying asset price is not assumed to follow a particular form like (1). That means the asset price could be a more general mean-reverting diffusion, a mean-reverting diffusion with jumps, a mean-reversion with regime switching, etc. As a result, parameters calibration is not required. The main idea of the approach is to construct a sequence of estimated threshold prices from an observed price at a certain time, followed by updating the threshold estimates using stochastic approximation algorithms. The performance of the algorithms is tested under real market data.

References

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