Math 2250 **Fall 2007**

Exam 2 - Practice Questions

1. Find $\frac{dy}{dx}$ for the following.

   (a) $y = xe^{-1/x}$

   (b) $y = \ln(\ln x)$

   (c) $xe^y = y - 1$

   (d) $y = \ln(x\sqrt{x^2+1})$

   (e) $y = \sqrt{x+1}\ (2-x)^5(x+3)^{-7}$

   (f) $y = \ln\left(\frac{1}{x}\right) + \frac{1}{\ln x}$

   (g) $y = \tan^{-1}(1-x^2)$

   (h) $x^2 = \sin^{-1} y$

2. Find an equation for the tangent line to the following curves at the indicated points.

   (a) $y = \ln(e^x + e^{2x})$ at $(0, \ln 2)$

   (b) $y = x\ln x$ at (e, e)

   (c) $y = xe^{2x}$ at $(1, e^2)$

   (d) $y = e^{2x}\cos x$ at $(0, 1)$

3. Find the maximum and minimum values attained by the given functions on the indicated closed intervals.

   (a) $f(x) = x^4 - 4x^3 + 4x^2 + 2$ on $[0, 4]$

   (b) $g(x) = x^{2/3}(5-x)$ on $[-1, 8]$

   (c) $h(x) = x + \frac{4}{x}$ on $[1, 4]$

4. Use implicit differentiation to find $\frac{dy}{dx}$ for the following implicit equations.

   (a) $y^2 + 3x + xy = 5$

   (b) $xy = -8$

   (c) $\dfrac{y}{x-y} = x^2 + 1$

   (d) $\sin(x+y) = y^2\cos x$

5. Determine the open intervals on the x-axis for which the given functions are increasing/decreasing.

   (a) $f(x) = x^3 + 2x^2 - x + 1$

   (b) $f(x) = x^2e^x$

   (c) $f(x) = \dfrac{\ln x}{\sqrt{x}}$

6. Sketch the graphs of the following functions.

   (a) $f(x) = \frac{x^2}{\sqrt{x+1}}$

   (b) $g(x) = \ln(4 - x^2)$

   (c) $h(x) = \frac{3x^5 - 20x^3}{32}$

   (d) $\ell(x) = \frac{\sqrt{x}(x-5)^2}{4}$

7. (a) Find two numbers whose sum is 100 and whose product is a maximum.

   (b) Find two numbers whose difference is 100 and whose product is a minimum.

8. (a) Find the points on the hyperbola $y^2 - x^2 = 4$ that are closest to the point $(2, 0)$.

   (b) Find the points on parabola $x - y^2 = 0$ that are closest to the point $(0, -3)$.

9. A 10 cm piece of wire is cut into two pieces. One piece is folded to make a square and the other to make a equilateral triangle. How should the wire be cut so that the total area enclosed is

   (a) maximized? (b) minimized?

10. A man is at point A on a bank of a straight canal, 3 km wide, and wants to reach point B, 8 km downstream on the opposite bank as quickly as possible. If he can row a boat at 8 km/h and run (on land) at 10 km/h, where should he row to?

11. A trapezoid is inscribed in a circle of radius 2. The long side of the trapezoid coincides with the diameter of the circle. What is the maximum possible area of such a trapezoid?

12. A flower bed of area 10π square feet is to be made in the shape of a sector of a circle. What is the minimum perimeter if such a bed?

13. Find the volume of the largest box that can be from a piece of cardboard 24 inches square by cutting equal squares from the corners and turning up the edges.

14. Gravel is being dumped from a conveyer belt at a rate of 30 cubic feet per min and the coarseness is such that it forms a pile in the shape of a cone whose base diameter and height are always equal. How fast is the height of the pile increasing when the pile is 10 feet high?

15. Two cars start moving from the same point. One travels south at 60 mph while the other travels west at 25 mph. At what rate are the distance between the cars increasing two hours?

16. Find the largest possible volume of a cylinder that can contained in a sphere of radius 10 cm.

17. Two straight roads in the desert intersect at right angles. A car is travelling at a constant speed of 75 mph down one road (speeding!). A quater of a mile from the intersection on the other road a stationary observer (the police) watches the car approach the intersection, how fast is the car approaching the observer (i.e. how fast does the police think the car is travelling) when the car is half a mile from the intersection?

18. A huge conical tank is to be made from a circular piece of sheet metal of radius 10 metres by cutting out a sector with vertex angle α and the welding together the remaining straight edges. Find α so that the resulting cone has maximum possible volume.