

Practice Final Exam

1. Find the derivative of the following functions.

(a) $f(x) = \ln\left(\frac{x^2 - 1}{x^2 + 1}\right)$

(b) $g(x) = \sin^2 x + \sin(x^2)$

(c) $F(x) = e^{x^2}(2x^2 - x)$

(d) $G(x) = x^x$

2. Find the equation of both tangent lines to the ellipse $x^2 + 4y^2 = 36$ that pass through the point $(12, 3)$

3. Use the definition of the derivative to find $f'(x)$ when $f(x) = \frac{1 + x^2}{x}$.

4. For what values of a is the function

$$f(x) = \begin{cases} 3x & \text{if } x < -2 \\ ax^2 & \text{if } x \geq -2 \end{cases}$$

continuous everywhere.

5. Calculate the following limits, do **not** use l'Hôpital's rule.

(a) $\lim_{x \rightarrow 0} \frac{\sin 3x}{2x}$

(b) $\lim_{x \rightarrow 0} \frac{\sin x}{x + \tan x}$

6. Use l'Hôpital's rule to calculate the following limits.

(a) $\lim_{x \rightarrow \infty} x^2 e^{-x}$

(b) $\lim_{x \rightarrow 0^+} (1 + x)^{1/x}$

7. Evaluate the following indefinite integrals.

(a)

$$\int \frac{\sec x \tan x}{1 + \sec x} dx$$

(b)

$$\int \frac{1}{\sqrt{9 - 4x^2}} dx$$

(c)

$$\int \frac{1}{\sqrt{x}(x+1)} dx$$

8. Use the definition of the definite Riemann integral to find

$$\int_0^2 (x^3 - 3x) dx.$$

9. Evaluate the following definite integrals.

(a)

$$\int_0^1 \frac{x}{\sqrt{9 - 4x^2}} dx$$

(b)

$$\int_1^2 x\sqrt{x-1} dx$$

(c)

$$\int_{-\pi/2}^{\pi/2} \frac{x^2 \sin x}{1 + x^6} dx$$

10. Find the area of the regions bounded by the curves $y = x^3$ and $y = x^{1/3}$.

11. Find $h'(x)$ if

$$h(x) = \int_{x^2}^x \frac{\sin t}{t} dt.$$

12. Find the maximum and minimum values attained by $f(x) = x^{2/3}(5 - x)$ on $[-1, 8]$.

13. Sketch the graph of $y = 3x^4 - 4x^3 - 12x^2 + 2$, indicate any asymptotes, local extrema or points of inflection.

14. Neil was just in Toronto and thought about dropping a stone from the observation deck (the Space Deck) of the CN Tower, 450 m above the ground. (Assume that the acceleration due to gravity in Canada is -10 m/s^2 .)

(a) How long would it take for the stone to reach the ground?

(b) With what velocity would it strike the ground?

(c) How would these answers change for a second stone thrown upwards with an initial velocity of 5 m/s ?

(d) How would these answers change for a third stone thrown with an initial velocity of 5 m/s at an angle of 30 degrees (above horizontal)? How far from the base of the tower would this stone land?

15. You are to design a cylindrical container with one end open and one end closed. The container is to hold 512π cubic inches. Find the dimensions of the cylinder which can be manufactured with the least amount of material.

16. The state police have a car parked 100 feet off the highway with a radar gun aimed at an angle of $\pi/4$ radians up the road. As a car approaches, radar indicates that it approaches the police car at 50 mph . The speed limit on the highway is 70 mph ; would the state police be justified in giving the driver a speeding ticket? Does it matter how far the parked police car is from the side of the road?