

## Exam 1

1. (20 points) Evaluate the following limits if they exist.

(a)  $\lim_{x \rightarrow 2} \frac{x^2 - x - 2}{x - 2}$

(b)  $\lim_{x \rightarrow 4} \frac{\sqrt{x+5} - 3}{x - 4}$

(c)  $\lim_{x \rightarrow 0} \frac{\tan 2x}{\sin 3x}$

(d)  $\lim_{x \rightarrow 0^-} \frac{2x}{3x - |x|}$

2. (20 points) In this question we consider the graph  $y = f(x)$  where

$$f(x) = \frac{2x^2 + 3x + 1}{(x+1)(x-1)}.$$

- (a) Find all points at which the function  $f(x)$  has a removable discontinuity.
- (b) Find all horizontal asymptotes of the graph  $y = f(x)$ .
- (c) Find all vertical asymptotes of the graph  $y = f(x)$ .
- (d) Find all points at which the tangent line to the curve  $y = f(x)$  is horizontal.
3. (20 points)

- (a) Use the definition of the derivative to find  $f'(x)$  when

$$f(x) = x^3 + 1.$$

- (b) Using part (a), or otherwise, find the equation of the tangent to the curve  $y = x^3 + 1$  at the point  $(2, 9)$ .
- (c) Find the points on the curve  $y = x^3 + 1$  where the tangent is parallel to the tangent in part (b).
4. (20 points) Find the derivative of the following functions.

(a)  $f(x) = x^5 + x^{1/5} + 5 + \frac{1}{x^{1/5}} + \frac{1}{x^5}$

(b)  $g(t) = \frac{1-t}{1+t}$

(c)  $h(x) = \cos^3 x$

(d)  $F(z) = \sqrt{1+z^2}$

5. (10 points) For what values of  $a$  is the function

$$g(x) = \begin{cases} \sin x & \text{if } x \leq 0 \\ ax & \text{if } x > 0 \end{cases}$$

- (a) continuous at  $x = 0$ ?
- (b) differentiable at  $x = 0$ ?

Be sure to justify your answers.

6. (10 points) State the *Intermediate Value Theorem* and use it to show that there exists a number whose square is one less than its cube. Find the best integer approximation for this number.