

Exam 2

1. (40 points) Find $\frac{dy}{dx}$ if
 - (a) $y = xe^{-x^2}$
 - (b) $x^3 + y^3 = 6xy$
 - (c) $y = \ln(\ln x)$
 - (d) $y = \frac{(x-6)^3\sqrt{x^2+1}}{(6-5x)^{1/5}}$
2. (20 points) A fence is to be built to enclose a rectangle of 300 square feet. The fence is to be made of material that cost 3 dollars per foot on 3 of its sides and material that cost 7 dollars per foot on the final side. Find the dimensions of the enclosure that is most economical to construct.
3. (20 points) Water is being pumped into a tank that has the shape of an inverted circular cone with base radius 2 metres and height 4 metres. At what rate must the water be being pumped in at if the water level is observed to be rising at a rate of 1 metre/min when the water is 2 metres deep?

Hint: The volume of a circular cone of radius r and height h is given by the formula

$$V = \frac{1}{3}\pi r^2 h.$$

4. (20 points) If $f(x) = x^4 - 4x^3 + 10$, then

$$f'(x) = 4x^2(x-3) \quad \text{and} \quad f''(x) = 12x(x-2).$$

- (a) Identify the intervals on which f increasing and decreasing.
- (b) Identify the intervals on which f concave up and concave down.
- (c) Use the information that you have obtained in parts (a) and (b) to **carefully** graph $y = f(x)$.

Be sure to indicate any local extrema or points of inflection.