

Exam 3 (Section 1)

1. (10 points) Solve the initial value problem

$$\frac{dy}{dx} = \frac{1}{x}, \quad y(2) = 1.$$

2. (20 points) Evaluate the following *indefinite integrals*.

(a)

$$\int \sin^2 x \, dx$$

(b)

$$\int \frac{x^3}{\sqrt{x^2 + 2}} \, dx$$

3. (20 points) **Recall that if f is continuous on $[a, b]$, then f is also integrable on $[a, b]$ and the definition of the definite integral simplifies to**

$$\int_a^b f(x) \, dx = \lim_{n \rightarrow \infty} R_n$$

where

$$R_n = \frac{b-a}{n} \sum_{k=1}^n f\left(a + k \frac{b-a}{n}\right).$$

- (a) Use this formula to evaluate the integral

$$\int_1^5 (x-1)^2 \, dx.$$

- (b) Check your answer to part (a) by instead evaluating the integral using the Fundamental Theorem of Calculus Part II.
 (c) What is the average value of the function $f(x) = (x-1)^2$ on the interval $[1, 5]$?

4. (10 points) Find $f'(x)$ when

$$f(x) = \int_1^{x^4} \ln t \, dt.$$

5. (20 points) Evaluate the following *definite integrals*.

(a)

$$\int_0^{\pi/2} 2 \cos^3 \theta \sin \theta \, d\theta$$

(b)

$$\int_1^e \frac{\ln x}{x} \, dx$$

6. (20 points)

- (a) Find the total area between the graph $y = \cos x$, the x -axis, and the vertical lines $x = 0$ and $x = 3\pi/4$.
 (b) Find the area of the region bounded by the line $4x - 3y = 4$ and the parabola $y^2 = 4x$.