Math 2250 **December 3, 2007**

Exam 3 (Section 2)

1. (10 points) Solve the initial value problem

$$\frac{dy}{dx} = \frac{1}{x}\,,\ y(2) = 1.$$

2. (20 points) Evaluate the following *indefinite integrals.*

   (a)
$$\int \cos^2 x\, dx$$

   (b)
$$\int \frac{x^5}{\sqrt{x^3+9}}\, dx$$

3. (20 points) **Recall that if f is continuous on $[a,b]$, then f is also integrable on $[a,b]$ and the definition of the definite integral simplifies to**
$$\int_a^b f(x)\, dx = \lim_{n\to\infty} R_n$$
where
$$R_n = \frac{b-a}{n}\sum_{k=1}^{n} f\Big(a + k\frac{b-a}{n}\Big).$$

   (a) Use this formula to evaluate the integral
$$\int_{-1}^{3} (x+1)^2\, dx.$$

   (b) Check your answer to part (a) by instead evaluating the integral using the Fundamental Theorem of Calculus Part II.

   (c) What is the average value of the function $f(x) = (x+1)^2$ on the interval $[-1,3]$?

4. (10 points) Find $f'(x)$ when
$$f(x) = \int_1^{x^4} \sec t\, dt.$$

5. (20 points) Evaluate the following *definite integrals.*

   (a)
$$\int_0^{\pi/2} 2\sin^3\theta\cos\theta\, d\theta$$

   (b)
$$\int_e^{e^2} \frac{1}{x\ln x}\, dx$$

6. (20 points)

   (a) Find the total area between the graph $y = \sin x$, the x-axis, and the vertical lines $x = -\pi/2$ and $x = \pi/4$.

   (b) Find the area of the regions bounded by the curves $y = -x$ and $y^2 = 2 - x$.