Math 2260 **Spring 2009**

Exam 2 - Practice Questions

ANSWERS

1. $\displaystyle\int \frac{2x+5}{x-3}\,dx = 2x + 11\ln|x-3| + C$ (rational function/substitution)
2. $\displaystyle\int x^2\sqrt{x-1}\,dx = \frac{2}{7}(x-1)^{7/2} + \frac{4}{5}(x-1)^{5/2} + \frac{2}{3}(x-1)^{3/2} + C$ (substitution)
3. $\displaystyle\int e^{x+e^x}\,dx = e^{e^x} + C$ (substitution)
4. $\displaystyle\int \frac{\cos\sqrt{x}}{\sqrt{x}}\,dx = 2\sin\sqrt{x} + C$ (substitution)
5. $\displaystyle\int x^2\sin 2x\,dx = -\frac{1}{2}x^2\cos 2x + \frac{1}{2}x\sin 2x + \frac{1}{4}\cos 2x + C$ (integration by parts)
6. $\displaystyle\int \sin^{-1} 2x\,dx = x\sin^{-1} 2x + \frac{1}{2}\sqrt{1-4x^2} + C$ (integration by parts)
7. $\displaystyle\int \tan^2 x\,dx = \tan x - x + C$ (convert to secants)
8. $\displaystyle\int \cos^4 2x\,dx = \frac{3}{8}x + \frac{1}{8}\sin 4x + \frac{1}{64}\sin 8x + C$ (half-angle identity)
9. $\displaystyle\int \frac{1}{x^2\sqrt{1-x^2}}\,dx = -\frac{\sqrt{1-x^2}}{x} + C$ (trigonometric substitution)
10. $\displaystyle\int \frac{x^2}{x+1}\,dx = \frac{1}{2}(x-3)(x+1) + \ln|x+1| + C$ (substitution)
11. $\displaystyle\int \frac{x^2}{(x+1)^3}\,dx = \ln|x+1| + \frac{2}{x+1} - \frac{1}{2(x+1)^2} + C$ (partial fractions)
12. $\displaystyle\int \frac{x^3}{x^2+1}\,dx = \frac{1}{2}x^2 - \frac{1}{2}\ln|x^2+1| + C$ (rational function)
13. $\displaystyle\int \frac{x^2}{x^3+1}\,dx = \frac{1}{3}\ln|x^3+1| + C$ (substitution)
14. $\displaystyle\int \frac{x^2+2}{x^3+1}\,dx = \ln|x+1| + \frac{2}{\sqrt{3}}\tan^{-1}\frac{2x-1}{\sqrt{3}} + C$ (partial fractions)
15. $\displaystyle\int \frac{e^x}{e^{2x}+1}\,dx = \tan^{-1}(e^x) + C$ (substitution)
16. $\displaystyle\int \ln x\,dx = x\ln x - x + C$ (integration by parts)

17. $\displaystyle\int \frac{1+x}{1+x^2}\,dx = \frac{1}{2}\ln|1+x^2| + \tan^{-1}x + C$ (rational function)

18. $\displaystyle\int \sqrt{x}\ln x\,dx = \frac{2}{3}x^{3/2}\ln x - \frac{4}{9}x^{3/2} + C$ (integration by parts)

19. $\displaystyle\int e^{2x}\cos 3x\,dx = \frac{1}{13}e^{2x}(2\cos 3x + 3\sin 3x) + C$ (integration by parts)

20. $\displaystyle\int x\sin x\cos x\,dx = \frac{1}{8}(\sin 2x - 2x\cos 2x) + C$ (integration by parts)

21. $\displaystyle\int (\ln x)^2\,dx = x(\ln x)^2 - 2x\ln x + 2x + C$ (integration by parts)

22. $\displaystyle\int x\sin^3(x^2)\,dx = \frac{1}{6}\cos^3(x^2) - \frac{1}{2}\cos(x^2) + C$ (substitution/trigonometric integral)

23. $\displaystyle\int \sin^2 x\cos^2 x\,dx = \frac{1}{8}x - \frac{1}{32}\sin 4x + C$ (half-angle identities)

24. $\displaystyle\int x\tan^{-1}x\,dx = \frac{1}{2}x^2\tan^{-1}x + \frac{1}{2}\tan^{-1}x - \frac{1}{2}x + C$ (integration by parts)

25. $\displaystyle\int \frac{x^2}{\sqrt{4x-x^2}}\,dx = 6\sin^{-1}\left(\frac{x-2}{2}\right) - 4\sqrt{4x-x^2} - \left(\frac{x-2}{2}\right)\sqrt{4x-x^2} + C$ (trig sub)

26. $\displaystyle\int \frac{1}{\sqrt{x^2+4x+8}}\,dx = \ln(\sqrt{x^2+4x+8} + x + 2) + C$ (complete square/trig sub)

27. $\displaystyle\int \frac{6x^2+5x-3}{x^3+2x^2-3x}\,dx = \ln|x| + 3\ln|x+3| + 2\ln|x-1| + C$ (partial fractions)

28. $\displaystyle\int \frac{\sin x - \cos x}{\sin x + \cos x}\,dx = -\ln|\sin x - \cos x| + C$ (substitution)

29. $\displaystyle\int \sin^2 2x\cos^3 2x\,dx = \frac{1}{6}\sin^3 2x - \frac{1}{10}\sin^5 2x + C$ (trigonometric integral)

30. $\displaystyle\int x^3\ln x\,dx = \frac{1}{4}x^4\ln x - \frac{1}{16}x^4 + C$ (integration by parts)

31. $\displaystyle\int \frac{\sqrt{x-2}}{x+2}\,dx = 2\sqrt{x-2} - 4\tan^{-1}(\frac{1}{2}\sqrt{x-2}) + C$ (substitution)

32. $\displaystyle\int \frac{\sqrt{1+\ln x}}{x\ln x}\,dx = 2\sqrt{1+\ln x} + \ln\left(\frac{\sqrt{1+\ln x}-1}{\sqrt{1+\ln x}+1}\right) + C$ (substitution)

33. $\displaystyle\int \tan^3 x\sec^4 x\,dx = \frac{1}{6}\tan^6 x + \frac{1}{4}\tan^4 x + C$ (trigonometric integral)

34. $\displaystyle\int \frac{x}{x^2+3x+2}\,dx = \ln(x+2)^2 - \ln|x+1| + C$ (partial fractions)

35. $\displaystyle\int \frac{x^3+x+1}{x^4+2x^2+4x}\,dx = \frac{1}{4}\ln|x^4+2x^2+4x| + C$ (substitution)

36. $\displaystyle\int \cos\sqrt{x}\,dx = 2\sqrt{x}\sin\sqrt{x} + 2\cos\sqrt{x} + C$ (substitution)

37. $\displaystyle\int e^{3x}\cos 5x\,dx = \frac{1}{34}e^{3x}(5\sin 5x + 3\cos 5x) + C$ (integration by parts)

38. $\displaystyle\int \frac{x}{1-x^2+\sqrt{1-x^2}}\,dx = -\ln(\sqrt{1-x^2}+1) + C$ (substitution)

39. $\displaystyle\int \frac{1+\cos x}{\sin x}\,dx = \ln|1-\cos x| + C$ (trigonometric integral)

40. $\displaystyle\int x^5 e^{-x^3}\,dx = -\frac{1}{3}e^{-x^2}(x^3+1) + C$ (integration by parts)

41. $\displaystyle\int \frac{e^{\tan^{-1}x}}{x^2+1}\,dx = e^{\tan^{-1}x} + C$ (substitution)

42. $\displaystyle\int \frac{1}{x^2-9}\,dx = \frac{1}{6}\ln\left|\frac{x-3}{x+3}\right| + C$ (partial fractions)

43. $\displaystyle\int (x+\sin x)^2\,dx = \frac{1}{3}x^3 + \frac{1}{2}x = 2\sin x - \frac{1}{2}\sin x\cos x - 2x\cos x + C$ (expand)

44. $\displaystyle\int \sin x\cos(\cos x)\,dx = -\sin(\cos x) + C$ (substitution)

45. $\displaystyle\int \frac{x\ln x}{\sqrt{x^2-1}}\,dx = \sqrt{x^2-1}(\ln x - 1) + \tan^{-1}\sqrt{x^2-1} + C$

46. $\displaystyle\int \sqrt{1+x-x^2}\,dx = \frac{5}{8}\sin^{-1}\left(\frac{1}{\sqrt{5}}(2x-1)\right) + \frac{1}{4}(2x-1)\sqrt{1+x-x^2} + C$

47. $\displaystyle\int \sqrt{\frac{1+x}{1-x}}\,dx = \sin^{-1}x - \sqrt{1-x^2} + C$

48. $\displaystyle\int \frac{1}{\sqrt{x+1}+\sqrt{x}}\,dx = \frac{2}{3}(x+1)^{3/2} - \frac{2}{3}x^{3/2} + C$

49. $\displaystyle\int \frac{1}{1+2e^x-e^{-x}}\,dx = \frac{1}{3}\ln\left|\frac{2e^x-1}{e^x+1}\right| + C$ (substitution)

50. $\displaystyle\int \frac{x^3+1}{x^3-x^2}\,dx = x + 2\ln|x-1| - \ln|x| - \frac{1}{x} + C$ (partial fractions)

51. $\displaystyle\int \frac{\ln(x+1)}{x^2}\,dx = \ln|x| - \left(1+\frac{1}{x}\right)\ln(x+1) + C$ (integration by parts)

52. $\displaystyle\int \frac{\sin 2x}{\sqrt{9-\cos^4 x}}\,dx = -\sin^{-1}(\frac{1}{2}\cos^2 x) + C$ (substitution)

53. $\displaystyle\int x\sqrt{2x-25}\,dx = \frac{1}{10}(2x-25)^{5/2} + \frac{25}{6}(2x-25)^{3/2} + C$ (substitution)

54. $\displaystyle\int \sec^3 x\,dx = \sec x\tan x + \ln|\sec x + \tan x| + C$ (integration by parts)