

## Exam 3 - Practice Questions

1. Determine whether the sequence converges or diverges. If it converges, find the limit.

$$(a) a_n = \frac{n \ln n}{n^2 + 1} \quad (b) b_n = (1 + 2n)^{1/n} \quad (c) c_n = \frac{\cos \sqrt{n}}{\sqrt{n}}$$

2. Test the series for convergence or divergence.

$$(a) \sum_{n=1}^{\infty} \frac{\sqrt{n}}{n^2 + 3} \quad (b) \sum_{n=0}^{\infty} \cos(n) \quad (c) \sum_{n=1}^{\infty} \frac{4^n}{3^{2n-1}} \quad (d) \sum_{n=2}^{\infty} \frac{(-1)^n}{(\ln n)^2}$$

$$(e) \sum_{n=1}^{\infty} \frac{2n}{8n - 5} \quad (f) \sum_{n=2}^{\infty} \frac{2}{n(\ln n)^3} \quad (g) \sum_{n=1}^{\infty} \frac{3^n n^2}{n!} \quad (h) \sum_{n=1}^{\infty} \frac{3^n}{5^n + n}$$

3. Test the series for convergence or divergence.

$$(a) \sum_{n=1}^{\infty} \sin(1/n) \quad (b) \sum_{n=1}^{\infty} n \sin(1/n) \quad (c) \sum_{n=1}^{\infty} \ln(1 + n^{-2})$$

4. Find the *radius of convergence* and *interval of convergence* of the power series.

$$(a) \sum_{n=0}^{\infty} \frac{x^n}{n + 3} \quad (b) \sum_{n=1}^{\infty} \frac{(-1)^n x^n}{n 2^n} \quad (c) \sum_{n=0}^{\infty} \frac{3^n x^n}{(n + 1)^2}$$

$$(d) \sum_{n=0}^{\infty} \frac{n}{4^n} (2x - 1)^n \quad (e) \sum_{n=0}^{\infty} (-1)^n \frac{(x - 1)^n}{\sqrt{n}} \quad (f) \sum_{n=0}^{\infty} \frac{(x - 2)^n}{n^n}$$

5. Find a power series representation for the function and determine the interval of convergence.

$$(a) f(x) = \frac{1}{4 + x^2} \quad (b) g(x) = \frac{1}{(1 + x)^2} \quad (c) h(x) = x \ln(1 + x)$$

6. Find the series' *interval of convergence* and, within this interval, the actual sum of the series as a function of  $x$ .

$$(a) \sum_{n=0}^{\infty} \frac{(x + 1)^{2n}}{9^n} \quad (b) \sum_{n=0}^{\infty} \left( \frac{x^2 - 1}{2} \right)^n \quad (c) \sum_{n=0}^{\infty} (-1)^n \frac{x^{4n}}{n!}$$

7. Find the Taylor series for the function  $f$  at  $a$ .

$$(a) \sin 2x, \quad a = 0$$

$$(b) 1 + x + x^2 + x^3, \quad a = 1$$

$$(c) x e^{-x}, \quad a = 0$$

8. Find the Taylor polynomial  $P_n(x)$  for the function  $f$  at  $a$ .

$$(a) \cos x, \quad a = \pi/6, \quad n = 3$$

$$(b) \tan x, \quad a = 0, \quad n = 4$$

$$(c) \sqrt{x}, \quad a = 9, \quad n = 3$$

9. For what values of  $x$  do the following polynomials approximate  $\sin x$  to within 0.01

$$(a) P_1(x) = x \quad (b) P_3(x) = x - x^3/6 \quad (c) P_5(x) = x - x^3/6 + x^5/120$$

10. How accurately does  $1 + x + x^2/2$  approximate  $e^x$  for  $-1 \leq x \leq 1$ ? Can you find a polynomial that approximates  $e^x$  to within 0.01 on this interval?