Math 2260 **April 16, 2009**

Exam 3

1. (15 points) Determine whether the sequence converges or diverges. If it converges, find the limit.
   (a) $a_n = 5 - (0.9)^n$
   (b) $b_n = \dfrac{1 + \sin n}{n}$
   (c) $c_n = n^{1/n}$

2. (12 points) Evaluate the sum of the series.
   (a)
   $$\sum_{n=1}^{\infty} \frac{2^{n+1}}{5^n}$$
   (b)
   $$\sum_{n=0}^{\infty} \frac{(-1)^n \pi^{2n+1}}{4^n (2n+1)!}$$

3. (28 points) Test the series for convergence or divergence. Be sure to justify your answer.
   (a)
   $$\sum_{n=1}^{\infty} \frac{n^2}{n^3 + 1}$$
   (b)
   $$\sum_{n=1}^{\infty} \sqrt{\frac{n-1}{n}}$$
   (c)
   $$\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^3 + 1}}$$
   (d)
   $$\sum_{n=1}^{\infty} \frac{n^3}{2^n}$$

4. (15 points) Find the *radius of convergence* and *interval of convergence* of the power series
   $$\sum_{n=0}^{\infty} \frac{(-3)^n x^{2n}}{n+1}$$

5. (15 points) Let $P_3(x)$ denote the third order Taylor polynomial centered at 1 of the function $f(x) = \ln x$.
   (a) Write down $P_3(x)$.
   (b) Give an estimate for how well $P_3(2)$ and $P_3(1.5)$ approximate $\ln 2$ and $\ln(1.5)$ respectively.

6. (15 points)
   (a) For approximately what values of $x > 0$ can you replace e^{-x} by $1 - x + x^2/2$ with an error of magnitude no greater than 0.01?
   (b) Find a polynomial that approximates e^{-x} for $0 < x \leq 1$ to within an accuracy of 0.01?