

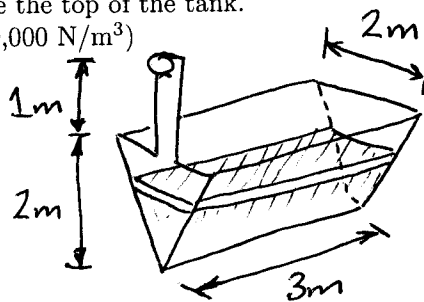
## Exam 1

1. (15 points) Solve the following two differential equations and in each case give a geometric description of the family of all solutions.

$$(a) \frac{dy}{dx} = -\frac{x}{y} \qquad (b) \frac{dy}{dx} = \frac{y}{x}, \quad x > 0, y > 0$$

2. (10 points) Find the volume of a napkin ring that is constructed by drilling a hole of radius 3 inches through the center of a sphere of radius 5 inches.
3. (20 points) Let  $\mathcal{R}$  denote the region in the plane bounded between the curves  $y = x^3$  and  $x = y^2$ .
- Set up, but do not evaluate, an integration in the variable  $y$  for the volume obtained by
    - revolving  $\mathcal{R}$  about the  $x$ -axis
    - revolving  $\mathcal{R}$  about the  $y$ -axis
  - Set up, but do not evaluate, an integration in the variable  $x$  for the volume obtained by
    - revolving  $\mathcal{R}$  about the line  $y = 1$
    - revolving  $\mathcal{R}$  about the line  $x = 1$
4. (10 points) A solid lies between planes perpendicular to the  $x$ -axis at  $x = 0$  and  $x = 1$ . The cross-sections perpendicular to the axis on the interval  $0 \leq x \leq 1$  are squares whose diagonals run from the curve  $y = x$  to the curve  $y = x^3$ . Set up, but do not evaluate, a definite integral for the volume of this solid.

5. (10 points) The tank below is full of water. Set up, but do not evaluate, a definite integral for the work required to empty the tank by pumping the water to a height of 1 meter above the top of the tank. (Assume that the water weighs  $10,000 \text{ N/m}^3$ )



6. (10 points) Find the length of the curve
- $$x = e^t \cos t, \quad y = e^t \sin t, \quad 0 \leq t \leq 1$$
7. (10 points) Find the area of the surface generated by rotating the curve

$$2y = 2 + x^2, \quad 1 \leq x \leq 2$$

about the  $y$ -axis.

8. (15 points)
- An empty bucket weighing 5 lb is lifted from the ground into the air by pulling in 20 ft of rope at a constant speed. The rope weighs 0.1 lb/ft. How much work was required to lift the bucket and rope?
  - How much work would be required to lift the same bucket and rope if the bucket instead starts by containing 10 lb of water, leaks water at a constant rate, and finishes draining just as it reaches the top?