**Math 2500** **Fall 2009**

# Exam 1 - Practice Questions

1. Find the unit tangent vector $\mathbf{T}(t)$ at the point with the given value of the parameter $t$.
   (a) $\mathbf{r}(t) = \langle 2t, 3t^2, 4t^3 \rangle, \quad t = 1$
   (b) $\mathbf{r}(t) = e^{2t} \cos t\, \mathbf{i} + e^{2t} \sin t\, \mathbf{j} + e^{2t}\, \mathbf{k}, \quad t = \pi/2$

2. Find parametric equations for the tangent line to the curve with the given parametric equations at the specified point.
   (a) $x = t$, $y = t^2$, $z = t^3$; $\quad (1, 1, 1)$
   (b) $x = t \cos 2\pi t$, $y = t \sin 2\pi t$, $z = 4t$; $\quad (0, 1/4, 1)$

3. Let $\mathbf{u}(t) = \mathbf{i} - 2t^2\, \mathbf{j} + 3t^3\, \mathbf{k}$ and $\mathbf{v}(t) = t\, \mathbf{i} + \cos t\, \mathbf{j} + \sin t\, \mathbf{k}$. Find
   (a) $\dfrac{d}{dt} [\mathbf{u}(t) \cdot \mathbf{v}(t)]$
   (b) $\dfrac{d}{dt} [\mathbf{u}(t) \times \mathbf{v}(t)]$

4. Show that if $\mathbf{r}$ is a vector function such that $\mathbf{r}''$ exists, then $\dfrac{d}{dt} [\mathbf{r}(t) \times \mathbf{r}'(t)] = \mathbf{r}(t) \times \mathbf{r}''(t)$.

5. Find the length of the given curve.
   (a) $\mathbf{r}(t) = \langle e^t, e^t \sin t, e^t \cos t \rangle, \quad 0 \le t \le 2\pi$
   (b) $\mathbf{r}(t) = t^2\, \mathbf{i} + 2t\, \mathbf{j} + \ln t\, \mathbf{k}, \quad 1 \le t \le e$

6. Reparametrize the curve with respect to arc-length measured from the point where $t = 0$ in the direction of increasing $t$.
   (a) $\mathbf{r}(t) = 3 \sin t\, \mathbf{i} + 4t\, \mathbf{j} + 3 \cos t\, \mathbf{k}$
   (b) $\mathbf{r}(t) = \cos^3 t\, \mathbf{i} + \sin^3 t\, \mathbf{j} + \cos 2t\, \mathbf{k}$

7. Find the curvature of the following curves.
   (a) $\mathbf{r}(t) = \langle \sin 4t, 3t, \cos 4t \rangle$
   (b) $\mathbf{r}(t) = t^2\, \mathbf{i} + 2t\, \mathbf{j} + \ln t\, \mathbf{k}$

8. Find the position vector of a particle that has the given acceleration and the given initial velocity and position.
   (a) $\mathbf{a}(t) = \mathbf{i} + 2\, \mathbf{j} + 2t\, \mathbf{k}, \quad \mathbf{v}(0) = \mathbf{0}, \quad \mathbf{r}(0) = 2\, \mathbf{i} + \mathbf{k}$
   (b) $\mathbf{a}(t) = t\, \mathbf{i} + t^2\, \mathbf{j} + \cos 2t\, \mathbf{k}, \quad \mathbf{v}(0) = \mathbf{i} + \mathbf{k}, \quad \mathbf{r}(0) = \mathbf{j}$

9. The position function of a spaceship is
$$\mathbf{r}(t) = (3 + t)\, \mathbf{i} + (2 + \ln t)\, \mathbf{j} + \left( 7 - \frac{4}{t^2 + 1} \right) \mathbf{k}$$
   and the coordinates of a space station are (6,4,9). The captain wants the spaceship to coast into the space station. When should the engines be turned off?

10. Find the indicated partial derivatives.
    (a) $f(x, y) = xe^{-y} + 3y$; $\quad f_y(1, 0)$
    (b) $f(x, y, z) = xyz$; $\quad f_y(0, 1, 2)$
    (c) $xy + yz = xz$; $\quad \partial z/\partial x$, $\partial z/\partial y$

11. Find all the second partial derivatives.

(a) $f(x, y) = x^2y + x\sqrt{y}$
(b) $f(x, y) = \sin(x + y) + \cos(x - y)$

12. (a) Verify that the function $u = e^{-a^2t} \sin x$ is a solution of the heat conduction equation $u_t = a^2u_{xx}$.
(b) Verify that the function $u = 1/\sqrt{x^2 + y^2 + z^2}$ is a solution of the three-dimensional Laplace equation $u_{xx} + u_{yy} + u_{zz} = 0$.

13. Find an equation of the tangent plane and normal line to the given surface at the specified point.

(a) $z = \ln(2x + y)$; $(-1, 3, 0)$
(b) $4x^2 + y^2 + z^2 = 24$; $(2, 2, 2)$
(c) $z + 1 = xe^y \cos z$; $(1, 0, 0)$

14. Let $f(x, y) = \sqrt{9x^2 + y^2}$.

(a) Find the linearization $L(x, y)$ of $f$ at the point (2,8).
(b) Find an approximate value for $\sqrt{9(1.95)^2 + (8.1)^2}$.

15. The length and width of a rectangle are measured as 30 cm and 24 cm, respectively, with an error in measurement of at most 0.1 cm in each. Estimate the maximum error in the calculated area of the rectangle.

16. Use the chain rule to find $dw/dt$.

(a) $w = \ln(x + y^2)$, $x = \sqrt{1 + t}$, $y = 1 + \sqrt{t}$
(b) $w = x/y + y/z$, $x = \sqrt{t}$, $y = \cos 2t$, $z = e^{-3t}$

17. Use the chain rule to find $\partial w/\partial s$ and $\partial w/\partial t$.

(a) $w = x^2 \sin y$, $x = s^2 + t^2$, $y = 2st$
(b) $w = x \arctan(xy)$, $x = t^2$, $y = se^t$

18. The radius of a right circular cylinder is decreasing at a rate of 1.2 cm/s while its height is increasing at a rate of 3 cm/s. At what rate is the volume of the cylinder changing when the radius is 80 cm and the height is 150 cm.

19. Find the directional derivative of the function at the given point in the direction of the vector $\mathbf{v}$.

(a) $f(x, y) = xe^{xy}$, $(-3, 0)$, $\mathbf{v} = 2\,\mathbf{i} + 3\,\mathbf{j}$
(b) $g(x, y, z) = \sqrt{xyz}$, $(2, 4, 2)$, $\mathbf{v} = 4\,\mathbf{i} + 2\,\mathbf{j} - 4\,\mathbf{k}$

20. Find the maximum rate of change of $f$ at the given point and the direction in which it occurs.

(a) $f(x, y) = xe^{-y} + 3y$, $(1, 0)$
(b) $f(x, y) = \cos(3x + 2y)$, $(\pi/6, -\pi/8)$

21. Find parametric equations for the tangent line to the curve of intersection of the paraboloid $z = x^2 + y^2$ and the ellipsoid $4x^2 + y^2 + z^2 = 9$ at the point (-1,1,2).

22. Find the local maximum and minimum values and saddle points of the function.

(a) $f(x, y) = x^2 + y^2 + 4x - 6y$
(b) $f(x, y) = xy - 2x - y$
(c) $f(x, y) = x^3 - 3xy + y^3$

23. Find the absolute maximum and minimum values of $f$ on the set $R$.

(a) $f(x, y) = 5-3x+4y$, and $R$ is the closed triangular region with vertices (0,0), (4,0), and (4,5)
(b) $f(x, y) = 1+xy-x-y$, and $R$ is the region bounded by the parabola $y = x^2$ and the line $y = 4$