Exam 2 - Practice Questions

1. Find the absolute maximum and minimum values of f on the set R.

   (a) $f(x,y) = 4xy^2 - x^2y^2 - xy^3$, and R is the closed triangular region with vertices (0,0), (0,6), and (6,0)

   (b) $f(x,y) = e^{-x^2-y^2}(x^2+2y^2)$, and R is the disk $x^2+y^2 \le 4$

2. Use Lagrange multipliers to find the maximum and minimum values of f subject to the given contraints.

   (a) $f(x,y) = \dfrac{1}{x} + \dfrac{1}{y}\,; \quad \dfrac{1}{x^2} + \dfrac{1}{y^2} = 1$

   (b) $f(x,y,z) = x+y+z\,; \quad \dfrac{1}{x} + \dfrac{1}{y} + \dfrac{1}{z} = 1$

3. Find the points on the surface $z^2 = xy + 1$ that are closest to the origin.

4. Find the dimensions of the rectangular box with largest volume if the total surface area is given by $64\,\text{cm}^2$.

5. Calculate the iterated integrals.

   (a)
   $$\int_{-2}^{2}\int_{0}^{4} (4x^3 + 3xy^2)\,dx\,dy$$

   (b)
   $$\int_{0}^{\pi}\int_{0}^{1} x\cos(xy)\,dy\,dx$$

   (c)
   $$\int_{0}^{1}\int_{0}^{y^2} e^{y^3}\,dx\,dy$$

   (d)
   $$\int_{0}^{1}\int_{0}^{x^2}\int_{0}^{y} y^2 z\,dz\,dy\,dx$$

6. (a) Describe the region whose area is given by the following integral and evaluate the integral
   $$\int_{0}^{\pi}\int_{1}^{1+\sin\theta} r\,dr\,d\theta$$

   (b) Describe the solid whose volume is given by the integral and evaluate the integral
   $$\int_{0}^{2\pi}\int_{0}^{\pi/6}\int_{1}^{3} \rho^2\sin\phi\,d\rho\,d\phi\,d\theta$$

7. Calculate the iterated integral by first reversing the order of integration.

   (a)
   $$\int_{0}^{1}\int_{x}^{1} e^{x/y}\,dy\,dx$$

   (b)
   $$\int_{0}^{1}\int_{y^2}^{1} y\sin(x^2)\,dx\,dy$$

8. Calculate the value of the multiple integral.

   (a) $\iint_R \frac{1}{(x-y)^2}\,dA$, where $R = \{(x,y) : 0 \le x \le 1,\ 2 \le y \le 4\}$

   (b) $\iint_R xy\,dA$, where R is bounded by $y^2 = x^3$ and $y = x$

   (c) $\iint_R (x^2+y^2)^{3/2}\,dA$, where R is the region in the first quadrant bounded by the lines $y = 0$ and $y = \sqrt{3}x$ and the circle $x^2 + y^2 = 9$

   (d) $\iiint_D x^2 z\,dV$, where $D = \{(x,y,z) : 0 \le x \le 2,\ 0 \le y \le 2x,\ 0 \le z \le x\}$

   (e) $\iiint_D yz\,dV$, where D lies above the plane $z = 0$, below the plane $z = y$, and inside the cyclinder $x^2 + y^2 = 4$

   (f) $\iiint_D z^3\sqrt{x^2+y^2+z^2}\,dV$, where D is the solid hemisphere with center the origin, radius 1, that lies above the xy-plane

9. Find the volume of the given solid.

   (a) Under the paraboloid $z = x^2 + 4y^2$ and above the rectangle $R = [0,2] \times [1,4]$

   (b) Under the surface $z = x^2 y$ and above the triangle in the xy-plane with vertices $(1,0)$, $(2,1)$ and $(4,0)$.

   (c) The solid tetrahedron with vertices $(0,0,0)$, $(0,0,1)$, $(0,2,0)$ and $(2,2,0)$

   (d) Bounded by the cylinder $x^2 + y^2 = 4$ and the planes $z = 0$ and $y + z = 3$

   (e) Above the paraboloid $z = x^2 + y^2$ and below the half-cone $z = \sqrt{x^2+y^2}$

10. Use polar coordinates to evaluate

$$\int_0^{\sqrt{2}} \int_y^{\sqrt{4-y^2}} \frac{1}{1+x^2+y^2}\,dx\,dy$$

11. Use spherical coordinates to evaluate

$$\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} (x^2+y^2+z^2)^2\,dz\,dy\,dx$$

12. Use cylinderical coordinates to evaluate

$$\int_{-1}^1 \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \int_{x^2+y^2}^{2-x^2-y^2} (x^2+y^2)^{3/2}\,dz\,dy\,dx$$

13. Express the integral $\iiint_D dV$ and an iterated integral in six different ways, where D is the solid bounded by the given surfaces.

   (a) $x^2 + z^2 = 4,\ y = 0,\ y = 6$

   (b) $z = 0,\ x = 0,\ y = 2,\ z = y - 2x$

   (c) $z = 0,\ z = y,\ x^2 = 1 - y$

14. Find the mass and center of mass of a thin plate that occupies the region bounded by the parabola $x = 1 - y^2$ and the coordinate axes in the first quadrant if the density function is $\delta(x,y) = y$.

15. Find the centroid of the solid that is bounded by the parabolic cylinder $z = 1 - y^2$ and the planes $x + z = 1$, $x = 0$ and $z = 0$.

16. Find the centroid of the solid that lies above the cone $\phi = \pi/3$ and below the sphere $\rho = 4\cos\phi$.