Math 2500 **Fall, 2009**

Final Exam - Practice Questions

1. You might remember from high school that the formula for the surface area of a cylinder of radius a and height h is $2\pi ah$.

   Obtain this formula by setting up and evaluating a surface integral.

2. Let $\mathbf{F}$ be the vector field

   $$\mathbf{F} = \langle z^2, x, -y^2 \rangle.$$

   Let S be the "cap" consisting of the part of the sphere $x^2+y^2+z^2 = 4$ which lies inside the cylinder $x^2 + y^2 = 1$. Compute the flux of curl $\mathbf{F}$

   $$\iint_S \operatorname{curl} \mathbf{F} \cdot \mathbf{n} \, d\sigma$$

   using any method you like.

3. For both parts (a) and (b) below, let $\mathbf{F}$ be the vector field

   $$\mathbf{F} = \langle e^x \sin y, e^x \cos y, xz \rangle.$$

   (a) Let S be the surface of the rectangular box bounded by the planes $x = 0$, $x = 1$, $y = 0$, $y = 1$, $z = 0$, and $z = 2$. Compute the flux

   $$\iint_S \mathbf{F} \cdot \mathbf{n} \, d\sigma$$

   using any method you like.

   (b) Now let S be the boundary of the region in the first octant enclosed by the sphere $x^2+y^2+z^2 = 1$ and the planes $x = 0$, $y = 0$, and $z = 0$. Compute the flux

   $$\iint_S \mathbf{F} \cdot \mathbf{n} \, d\sigma$$

   using any method you like.

4. Let $\mathbf{F}$ be the vector field

   $$\mathbf{F} = \langle 0, x, 0 \rangle.$$

   Let C be the curve which is the intersection of the surfaces $x^2+y^2 = 1$ and $z = xy$.

   Find the circulation of $\mathbf{F}$ around C by any method you like.

5. Is the vector field

   $$\mathbf{F} = \langle 2xy, x^2 - z^2, -2yz \rangle$$

   conservative? If so, find a potential function for $\mathbf{F}$. If not, explain how you know $\mathbf{F}$ is not conservative.

6. Let

$$f(x,y) = 2x^3 + xy^2 + 5x^2 + y^2.$$

with domain all of $\mathbb{R}^2$. Find all the critical points of f. For each critical point, determine whether the critical point is a local maximum, a local minimum, or a saddle point.

7. (a) Use Lagrange multipliers to find the maxima and minima of the function

$$f(x,y) = 2x + 4y$$

subject to the constraint

$$x^2 + y^2 = 4.$$

Note: There are other valid methods for solving this problem, but to get full credit, you must use Lagrange multipliers.

(b) Graph the function and the constraint. What is the geometric interpretation of your answer to part (a)?

(c) Let S be the portion of

$$z = 2x + 4y$$

which lies inside

$$x^2 + y^2 = 4.$$

i. Find the area of S using any method you like.

ii. Find the flux of $\mathbf{F} = \langle y, x, xy \rangle$ upwards across S.

8. Find an equation for the tangent plane to the surface

$$xy + yz + xz = 3$$

at the point (1,1,1).

9. *Interpreting gradients.*

(a) Compute the gradient of

$$f(x,y) = \sqrt{1 - x^2 - y^2}$$

at the point $(\frac{1}{2}, \frac{1}{2})$.

(b) Compute the gradient of

$$F(x,y,z) = x^2 + y^2 + z^2$$

at the point $(\frac{1}{2}, \frac{1}{2}, \sqrt{\frac{1}{2}})$.

(c) What is the relationship between the calculations you did in parts (a) and (b)? Are the answers the same? If they are the same, explain what they are telling you. If they are different, explain what each of them tells you.