Math 3100 **Fall 2005**

Practice Exam 2

1. (30 points) For each of the following series state whether it converges absolutely, converges conditionally, or diverges. Justify your answers.

   (a) $\sum_{n=1}^{\infty} \frac{\cos n}{2^n}$

   (b) $\sum_{n=2}^{\infty} \frac{(-1)^n n}{\sqrt{n^2+1}}$

   (c) $\sum_{n=1}^{\infty} \frac{(-2)^n}{n!}$

   (d) $\sum_{n=1}^{\infty} \frac{\ln(n)}{n^2}$

   (e) $\sum_{n=1}^{\infty} \frac{(-3)^n}{n^5}$

   (f) $\sum_{n=1}^{\infty} (-1)^n \sin(1/n^2)$

2. (15 points) Prove that if $\{a_n\}$ is summable, then $\lim_{n\to\infty} a_n = 0$.

3. (10 points) Prove that if $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ converge, then necessarily $\sum_{n=1}^{\infty}(a_n + b_n)$ converges, and moreover

$$\sum_{n=1}^{\infty}(a_n + b_n) = \sum_{n=1}^{\infty} a_n + \sum_{n=1}^{\infty} b_n$$

4. (10 points) For which values of x do the following series converge?

   (a) $\sum_{n=1}^{\infty} \frac{(2x)^n}{2n+1}$

   (b) $\sum_{n=1}^{\infty} \frac{(x-1)^n n}{2^n}$

5. (10 points) Prove that omitting or changing the the first few terms of a series does not affect its convergence.

6. (10 points) Evaluate

   (a) $\sum_{n=0}^{\infty} \frac{1}{5^n}$

   (b) $\sum_{n=3}^{\infty} \frac{2}{5^n}$

7. (15 points) Provide counterexamples to the following false statements:

   (a) If $\lim_{n\to\infty} a_n = 0$, then $\sum_{n=1}^{\infty} a_n$ converges.

   (b) If $\sum_{n=1}^{\infty} b_n$ converges, then $\sum_{n=1}^{\infty} |b_n|$ converges.

   (c) If $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = 1$, then $\sum_{n=1}^{\infty} |a_n|$ diverges.

Bonus question: Does the following series converge? What if you raise the sequence of terms to the kth power?

$$\sum_{n=1}^{\infty} \frac{1 \cdot 3 \cdot 5 \cdot 7 \cdots (2n-1)}{2 \cdot 4 \cdot 6 \cdots (2n)}$$