Math 3100 **Fall 2009**

Exam 2

No calculators. Show your work. Give full explanations. Good luck!

1. (20 points) For each of the following series state whether it converges absolutely, converges conditionally, or diverges. Justify your answers.

   (a) $\displaystyle\sum_{n=2}^{\infty}(-1)^n\frac{2n^2+1}{3n^2-1}$ (b) $\displaystyle\sum_{n=1}^{\infty}\frac{(-1)^n}{n^2+n}$

   (c) $\displaystyle\sum_{n=1}^{\infty} n\left(\frac{3}{4}\right)^n$ (d) $\displaystyle\sum_{n=3}^{\infty}\frac{(-1)^n}{n\ln n}$

2. (20 points) For which values of x do the following series converge?

   (a) $\displaystyle\sum_{n=1}^{\infty}\frac{(3x)^n}{n^2}$ (b) $\displaystyle\sum_{n=1}^{\infty}\frac{(x-2)^n}{n4^n}$

3. (20 points)

   (a) Find a closed form for the power series $\sum_{n=2}^{\infty} x^{2n}$.

   (b) Find a sequence $\{a_n\}$ so that $\sum_{n=0}^{\infty} a_n x^n = \dfrac{1}{4+x}$ for all $x \in (-4,4)$.

4. (15 points) Prove that if the sequence $\{a_n\}$ is summable, then $\lim_{n\to\infty} a_n = 0$.

5. (25 points)

   (a) State the *Property of Completeness.*

   (b) Let $\{a_n\}$ be a sequence of non-negative terms. Use the Property of Completeness to prove that $\sum a_n$ converges if and only if the sequence of partial sums is bounded. *(Be sure to prove both implications)*

   (c) Prove the Comparison Test: If $0 \le a_n \le b_n$ for all $n \in \mathbb{N}$ and that $\sum b_n$ is convergent, then $\sum a_n$ is also convergent.

Bonus question: Does the following series converge?

$$\sum_{n=1}^{\infty}\frac{1\cdot 3\cdot 5\cdot 7\cdots(2n-1)}{2\cdot 4\cdot 6\cdots(2n)}$$

More generally, for what values of p does the following series converge?

$$\sum_{n=1}^{\infty}\left(\frac{1\cdot 3\cdot 5\cdot 7\cdots(2n-1)}{2\cdot 4\cdot 6\cdots(2n)}\right)^p$$