Math 3100 **Fall 2009**

Exam 2 - Practice Questions

1. For each of the following series state whether it converges absolutely, converges conditionally, or diverges. Justify your answers.

   (a) $\sum_{n=1}^{\infty} \dfrac{\cos n}{2^n}$

   (b) $\sum_{n=2}^{\infty} \dfrac{(-1)^n n}{\sqrt{n^2+1}}$

   (c) $\sum_{n=1}^{\infty} \dfrac{(-2)^n(2n+1)}{n!}$

   (d) $\sum_{n=1}^{\infty} \dfrac{\ln(n)}{n^2}$

   (e) $\sum_{n=1}^{\infty} \dfrac{(-3)^n}{n^5}$

   (f) $\sum_{n=1}^{\infty}(-1)^n \sin(1/n^2)$

2. For what values of p does $\sum\limits_{n=1}^{\infty} \dfrac{1}{n(\ln n)^p}$ converge? Justify your answer.

3. For which values of x do the following series converge?

   (a) $\sum_{n=1}^{\infty} \dfrac{(2x)^n}{2n+1}$

   (b) $\sum_{n=1}^{\infty} \dfrac{(x-1)^n n}{2^n}$

4. (a) Evaluate the series $\sum\limits_{n=2}^{\infty} 4^{-n}$

   (b) Find a sequence $\{a_n\}$ so that $\sum\limits_{n=0}^{\infty} a_n x^n = \dfrac{1}{4-x}$ for all $x \in (-4, 4)$.

5. Provide counterexamples to the following false statements:

   (a) If $\lim\limits_{n\to\infty} a_n = 0$, then $\sum_{n=1}^{\infty} a_n$ converges.

   (b) If $\sum_{n=1}^{\infty} b_n$ converges, then $\sum_{n=1}^{\infty} |b_n|$ converges.

   (c) If $\lim\limits_{n\to\infty} \left|\dfrac{a_{n+1}}{a_n}\right| = 1$, then $\sum_{n=1}^{\infty} |a_n|$ diverges.

6. Prove that a sequence $\{a_n\}$ is Cauchy if and only if it is convergent.

7. Prove that if $\{a_n\}$ is summable, then $\lim\limits_{n\to\infty} a_n = 0$.

8. Prove that omitting or changing the the first few terms of a series does not affect its convergence.

9. Prove that if $a_n > 0$, $b_n > 0$, and $\lim\limits_{n\to\infty} \dfrac{a_n}{b_n} = 1$, then

$$\sum_{n=1}^{\infty} a_n \text{ convergent} \iff \sum_{n=1}^{\infty} b_n \text{ convergent.}$$

10. State and prove the ratio test for series of non-negative terms.