

Exam 3

Show your work. Give full explanations. Good luck!

1. (30 points)
 - (a) Carefully state the definition of uniform convergence of a sequence of functions $\{f_n\}$ to a function f on a set A .
 - (b) Prove that the sequence given by $f_n(x) = xe^{-nx}$ converges uniformly to zero on the interval $[0, \infty)$.
 - (c) Prove that that the sequence given by $f_n(x) = nxe^{-nx}$ does not converge uniformly to zero on the interval $[0, \infty)$.

2. (15 points) Assume that $\{f_n\}$ is a sequence of continuous functions converging uniformly to the function f on the interval $[a, b]$. Prove that f must be continuous on $[a, b]$.

3. (15 points)

- (a) Find the sixth order Macluarin polynomial for the function

$$f(x) = e^{-x} \sin x$$

- (b) Use part (a) to find (without differentiating) the value of $f^{(6)}(0)$.

4. (20 points) Let $P_3(x)$ denote the third order Taylor polynomial centered at 1 of the function $f(x) = \ln x$.

- (a) Find $P_3(x)$.
- (b) Give an estimate for how well $P_3(2)$ and $P_3(1.5)$ approximate $\ln 2$ and $\ln(1.5)$ respectively.

5. (20 points)

- (a) State Taylor's Theorem and prove that the $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$ for all $x \in \mathbb{R}$.
- (b) How accurately does the polynomial

$$1 + x + x^2/2 + x^3/6$$

approximate e^x for $|x| \leq 1/2$?

6. (Bonus points) For approximately what values of x can you replace the function

$$F(x) = \int_0^x \tan^{-1} t dt$$

by its forth order Macluarin polynomial with an error of magnitude less than 10^{-3} ?