Math 2200 - Practice Final Exam

Problem 1. Evaluate the derivative of the following functions.

a) $f(x) = \dfrac{1}{\frac{1}{x} - \frac{1}{x^2}}$

b) $f(x) = ln(e^{x^2+1})$ (simplify your answer)

c) $f(x) = x^{cos\, x}$

Problem 2. A cubical block of ice is melting in such a way that its surface area is decreasing at a constant rate $\frac{dS}{dt} = -5\ cm^2/s$.

(a) At what rate its edge is decreasing, when its edge is $r = 10\ cm$?

(b) At what rate its volume V is decreasing, when its edge is $r = 10\ cm$.

(c)* Now, suppose that the ice cube is melting in such a way that the rate of decrease of its volume is proportional to half of its surface area. At 10 am its volume is 1000 cm^3 and at $11am$ its volume is 125 cm^3.
When does the ice cube finish melting ?

Problem 3. Find the minimum and the maximum of the following functions on the given intervals.
Explain your answer, that is find all critical points, and evaluate your function at all points necessary.

a) $f(x) = \frac{x}{x^2+1}$ on [0,4]

b) $f(x) = |x + 1| + 2|x - 1|$ on [-2,2]

c) $f(x) = x(2 - x)^{1/3}$ on [1,3]

Problem 4.

a) Find the dimensions of the rectangle (with sides parallel to the coordinate axis) of maximal area that can be inscribed in the ellipse given by the equation

$$\frac{x^2}{4} + \frac{y^2}{9} = 1$$

b) Find the dimensions of the cylinder of maximal volume that can be inscribed in a sphere of radius 9, by completing the following steps

i) Find the equation which expresses the relation between the radius r of the bottom and the height h of the cylinder.

ii) Express the volume V of the cylinder as a function of the radius r.

i3) Find the critical points and the domain of the function you obtained in part b).

Note: The volume and the surface of a cylinder are given by: $V = \pi h r^2$, $S = \pi r^2 + 2\pi h r$.

Problem 5. A rocket is launched vertically from a point 10 mi west of an observer on the ground. If the speed of the rocket is 1500 mph, find the rate of increase of the angle of elevation of the observer's line of sight to the rocket, at the moment when the rocket is 10 mi above the ground.

Note: Convert the angles into radians, and find the relation between the angle of elevation and the height of the rocket.

Problem 6. The radius of a ball is measured as 10 in.

a) Suppose radius is measured with a maximum error of 0.1 in. What is the resulting maximal error in the calculated volume

b) With what accuracy of the radius of the ball be measured to ensure an error of at most 0.1 in^3 in its calculated volume ?

Problem 7. Consider the function: $f(x) = 2ln\,(1 + x) - x$ defined for $x > 0$.

a) Use the mean value theorem to show that the equation $f(x) = 0$ has exactly one solution.

b) Use the fist derivative test to find the global maximum of the function $f(x)$.

c) Use the second derivative test to find the global maximum of the function $f(x)$.

Problem 8.

a) Sketch the graph of the following function: $f(x) = \frac{ln\,x}{x}$.

That is, find the critical points, the intervals where the function is increasing and decreasing.

Then find the inflection points, the intervals where the function is convex or concave, and describe its behavior as x tends to infinity or 0.

Plot and label the intercepts, possible critical points and inflection points, and finally sketch the graph as accurately as you can.

b) Sketch the graph of the function: $f(x) = \frac{e^x}{e^x+1}$ by completing the steps as in part a).